Induction heuristics

Basic heuristics

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Induction on argument number i of f
if f is defined by recursion on argument number i

A tail recursive reverse

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A tail recursive reverse

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$itrev\ []\qquad ys = ys\ \ |$

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lemma *itrev xs [] = rev xs*

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Why in this direction?

A tail recursive reverse

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lemma $itrev\ xs\ [] = rev\ xs$

Why in this direction?

Because the lhs is “more complex” than the rhs.

Demo: first proof attempt

Generalisation (1)

Replace constants by variables

lemma *itrev xs ys = rev xs @ ys*

Demo: second proof attempt

Generalisation (2)

Quantify free variables by $\forall$
(except the induction variable)

lemma $\forall$ *ys. itrev xs ys = rev xs @ ys*