Magical Methods (Computing with Natural Numbers)

A book about Vedic mathematics describes three methods to make the calculation of squares of natural numbers easier:

- **MM1**: Numbers whose predecessors have squares that are known or can easily be calculated. For example:
  - Needed: $61^2$
  - Given: $60^2 = 3600$
  - Observe: $61^2 = 3600 + 60 + 61 = 3721$

- **MM2**: Numbers greater than, but near 100. For example:
  - Needed: $102^2$
  - Let $h = 102 - 100 = 2$, $h^2 = 4$
  - Observe: $102^2 = (102 + h)$ shifted two places to the left $+h^2 = 10404$

- **MM3**: Numbers ending in 5. For example:
  - Needed: $85^2$
  - Observe: $85^2 = (8*9)$ appended to $25 = 7225$
  - Needed: $995^2$
  - Observe: $995^2 = (99*100)$ appended to $25 = 990025$

In this exercise we will show that these methods are not so magical after all!

- Based on **MM1** define a function $sq$ that calculates the square of a natural number.

- Prove the correctness of $sq$ (i.e. $sq \ n = n \ * \ n$).

- Formulate and prove the correctness of **MM2**.
  - Hints:
    - Generalise **MM2** for an arbitrary constant (instead of 100).
    - Universally quantify all variables other than the induction variable.

- Formulate and prove the correctness of **MM3**.
  - Hints:
– Try to formulate the property ‘numbers ending in 5’ such that it is easy to get to the rest of the number.

– Proving the binomial formula for \((a + b)^2\) can be of some help.