Searching in Lists

Define a function \texttt{first\_pos} that computes the index of the first element in a list that satisfies a given predicate:

\[ \texttt{first\_pos} : \mathcal{P} \Rightarrow \text{nat} \]

The smallest index is \texttt{0}. If no element in the list satisfies the predicate, the behaviour of \texttt{first\_pos} should be as described below.

Verify your definition by computing

- the index of the first number equal to \texttt{3} in the list \texttt{[1:nat, 3, 5, 3, 1]},
- the index of the first number greater than \texttt{4} in the list \texttt{[1:nat, 3, 5, 7]},
- the index of the first list with more than one element in the list \texttt{[[], [1, 2], [3]].}

Note: Isabelle does not know the operators \texttt{> and \geq}. Use \texttt{< and \leq} instead.

Prove that \texttt{first\_pos} returns the length of the list if and only if no element in the list satisfies the given predicate.

Now prove:

\texttt{lemma "list\_all (\lambda x. \neg P x) (take (first\_pos P xs) xs)"}

How can \texttt{first\_pos (\lambda x. P x \lor Q x) xs} be computed from \texttt{first\_pos P xs} and \texttt{first\_pos Q xs}? Can something similar be said for the conjunction of \texttt{P} and \texttt{Q}? Prove your statement(s).

Suppose \texttt{P} implies \texttt{Q}. What can be said about the relation between \texttt{first\_pos P xs} and \texttt{first\_pos Q xs}? Prove your statement.

Define a function \texttt{count} that counts the number of elements in a list that satisfy a given predicate.

\[ \texttt{count} : \mathcal{P} \Rightarrow \text{nat} \]

Show: The number of elements with a given property stays the same when one reverses a list with \texttt{rev}. The proof will require a lemma.

Find and prove a connection between the two functions \texttt{filter} and \texttt{count}. 