Replace, Reverse and Delete

Define a function \texttt{replace}, such that \texttt{replace \ x \ y \ zs} yields \texttt{zs} with every occurrence of \texttt{x} replaced by \texttt{y}.

\texttt{consts replace :: "'a ⇒ 'a ⇒ 'a list ⇒ 'a list"}

Prove or disprove (by counterexample) the following theorems. You may have to prove some lemmas first.

\texttt{theorem "rev(replace \ x \ y \ zs) = replace \ x \ y \ (rev \ zs)"}
\texttt{theorem "replace \ x \ y \ (replace \ u \ v \ zs) = replace \ u \ v \ (replace \ x \ y \ zs)"}
\texttt{theorem "replace \ y \ z \ (replace \ x \ y \ zs) = replace \ x \ z \ zs"}

Define two functions for removing elements from a list: \texttt{del1} \texttt{ x xs} deletes the first occurrence (from the left) of \texttt{x} in \texttt{xs}, \texttt{delall} \texttt{ x xs} all of them.

\texttt{consts del1 :: "'a ⇒ 'a list ⇒ 'a list"}
\texttt{delall :: "'a ⇒ 'a list ⇒ 'a list"}

Prove or disprove (by counterexample) the following theorems.

\texttt{theorem "del1 \ x \ (delall \ x \ xs) = delall \ x \ xs"}
\texttt{theorem "delall \ x \ (delall \ x \ xs) = delall \ x \ xs"}
\texttt{theorem "delall \ x \ (delall \ y \ zs) = delall \ y \ (delall \ x \ zs)"}
\texttt{theorem "delall \ x \ (delall \ y \ zs) = delall \ y \ (delall \ x \ zs)"}
\texttt{theorem "delall \ y \ (replace \ x \ y \ xs) = delall \ x \ xs"}
\texttt{theorem "delall \ y \ (replace \ x \ y \ xs) = delall \ x \ xs"}
\texttt{theorem "replace \ \ x \ y \ (delall \ x \ zs) = delall \ x \ zs"}
\texttt{theorem "replace \ x \ y \ (delall \ z \ zs) = delall \ z \ (replace \ x \ y \ zs)"}
\texttt{theorem "rev(del1 \ x \ xs) = del1 \ x \ (rev \ xs)"}
\texttt{theorem "rev(delall \ x \ xs) = delall \ x \ (rev \ xs)"}