

# Isabelle/HOL Exercises

## Logic and Sets

### A Riddle: Rich Grandfather

First prove the following formula, which is valid in classical predicate logic, informally with pen and paper. Use case distinctions and/or proof by contradiction.

*If every poor man has a rich father,  
then there is a rich man who has a rich grandfather.*

**theorem**

$$\begin{aligned} & \text{"}\forall x. \neg \text{rich } x \longrightarrow \text{rich } (\text{father } x) \implies \\ & \exists x. \text{rich } (\text{father } (\text{father } x)) \wedge \text{rich } x\text{"} \end{aligned}$$

Now prove the formula in Isabelle using a sequence of rule applications (i.e. only using the methods *rule*, *erule* and *assumption*).