

# Isabelle/HOL Exercises

## Logic and Sets

### Predicate Logic

We are again talking about proofs in the calculus of Natural Deduction. In addition to the rules given in the exercise “Propositional Logic”, you may now also use

*exI*:  $P\ x \Longrightarrow \exists x. P\ x$

*exE*:  $[\exists x. P\ x; \bigwedge x. P\ x \Longrightarrow Q] \Longrightarrow Q$

*allI*:  $(\bigwedge x. P\ x) \Longrightarrow \forall x. P\ x$

*allE*:  $[\forall x. P\ x; P\ x \Longrightarrow R] \Longrightarrow R$

Give a proof of the following propositions or an argument why the formula is not valid:

**lemma** " $(\exists x. \forall y. P\ x\ y) \longrightarrow (\forall y. \exists x. P\ x\ y)$ "

**lemma** " $(\forall x. P\ x \longrightarrow Q) = ((\exists x. P\ x) \longrightarrow Q)$ "

**lemma** " $((\forall x. P\ x) \wedge (\forall x. Q\ x)) = (\forall x. (P\ x \wedge Q\ x))$ "

**lemma** " $((\forall x. P\ x) \vee (\forall x. Q\ x)) = (\forall x. (P\ x \vee Q\ x))$ "

**lemma** " $((\exists x. P\ x) \vee (\exists x. Q\ x)) = (\exists x. (P\ x \vee Q\ x))$ "

**lemma** " $(\forall x. \exists y. P\ x\ y) \longrightarrow (\exists y. \forall x. P\ x\ y)$ "

**lemma** " $(\neg (\forall x. P\ x)) = (\exists x. \neg P\ x)$ "