

# Isabelle/HOL Exercises

## Logic and Sets

### Propositional Logic

In this exercise, we will prove some lemmas of propositional logic with the aid of a calculus of natural deduction.

For the proofs, you may only use

- the following lemmas:

*notI*:  $(A \Longrightarrow \text{False}) \Longrightarrow \neg A$ ,

*notE*:  $[\neg A; A] \Longrightarrow B$ ,

*conjI*:  $[[A; B] \Longrightarrow A \wedge B$ ,

*conjE*:  $[A \wedge B; [[A; B] \Longrightarrow C]] \Longrightarrow C$ ,

*disjI1*:  $A \Longrightarrow A \vee B$ ,

*disjI2*:  $A \Longrightarrow B \vee A$ ,

*disjE*:  $[A \vee B; A \Longrightarrow C; B \Longrightarrow C] \Longrightarrow C$ ,

*impI*:  $(A \Longrightarrow B) \Longrightarrow A \longrightarrow B$ ,

*impE*:  $[A \longrightarrow B; A; B \Longrightarrow C] \Longrightarrow C$ ,

*mp*:  $[A \longrightarrow B; A] \Longrightarrow B$

*iffI*:  $[A \Longrightarrow B; B \Longrightarrow A] \Longrightarrow A = B$ ,

*iffE*:  $[A = B; [A \longrightarrow B; B \longrightarrow A] \Longrightarrow C] \Longrightarrow C$

*classical*:  $(\neg A \Longrightarrow A) \Longrightarrow A$

- the proof methods *rule*, *erule* and *assumption*.

Prove:

**lemma** *I*: " $A \longrightarrow A$ "

**lemma** " $A \wedge B \longrightarrow B \wedge A$ "

**lemma** " $(A \wedge B) \longrightarrow (A \vee B)$ "

**lemma** " $((A \vee B) \vee C) \longrightarrow A \vee (B \vee C)$ "

**lemma** *K*: " $A \longrightarrow B \longrightarrow A$ "

**lemma** " $(A \vee A) = (A \wedge A)$ "

**lemma** *S*: " $(A \longrightarrow B \longrightarrow C) \longrightarrow (A \longrightarrow B) \longrightarrow A \longrightarrow C$ "

**lemma** " $(A \longrightarrow B) \longrightarrow (B \longrightarrow C) \longrightarrow A \longrightarrow C$ "

**lemma** " $\neg \neg A \longrightarrow A$ "

**lemma** " $A \longrightarrow \neg \neg A$ "

**lemma** " $(\neg A \longrightarrow B) \longrightarrow (\neg B \longrightarrow A)$ "

**lemma** " $(A \longrightarrow B) \longrightarrow A \longrightarrow A$ "

**lemma** " $A \vee \neg A$ "

**lemma** " $\neg (A \wedge B) = (\neg A \vee \neg B)$ "