

# What's in Main

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## Abstract

This document lists the main types, functions and syntax provided by theory *Main*. It is meant as a quick overview of what is available. For infix operators and their precedences see the final section. The sophisticated class structure is only hinted at. For details see <http://isabelle.in.tum.de/library/HOL/>.

## HOL

The basic logic:  $x = y$ , *True*, *False*,  $\neg P$ ,  $P \wedge Q$ ,  $P \vee Q$ ,  $P \longrightarrow Q$ ,  $\forall x. P$ ,  $\exists x. P$ ,  $\exists!x. P$ , *THE*  $x. P$ .

*undefined* :: 'a  
*default* :: 'a

## Syntax

$x \neq y$   $\equiv \neg (x = y)$  ( $\sim =$ )  
 $P \longleftrightarrow Q$   $\equiv P = Q$   
*if*  $x$  *then*  $y$  *else*  $z$   $\equiv$  *If*  $x$   $y$   $z$   
*let*  $x = e_1$  *in*  $e_2$   $\equiv$  *Let*  $e_1$  ( $\lambda x. e_2$ )

## Orderings

A collection of classes defining basic orderings: preorder, partial order, linear order, dense linear order and wellorder.

*op*  $\leq$  :: 'a  $\Rightarrow$  'a  $\Rightarrow$  bool ( $\leq$ )  
*op*  $<$  :: 'a  $\Rightarrow$  'a  $\Rightarrow$  bool  
*Least* :: ('a  $\Rightarrow$  bool)  $\Rightarrow$  'a  
*min* :: 'a  $\Rightarrow$  'a  $\Rightarrow$  'a  
*max* :: 'a  $\Rightarrow$  'a  $\Rightarrow$  'a

$top \quad \quad \quad :: 'a$   
 $bot \quad \quad \quad \quad :: 'a$   
 $mono \quad \quad \quad \quad :: ('a \Rightarrow 'b) \Rightarrow bool$   
 $strict\_mono :: ('a \Rightarrow 'b) \Rightarrow bool$

## Syntax

$x \geq y \quad \quad \equiv \quad y \leq x \quad \quad \quad (>=)$   
 $x > y \quad \quad \equiv \quad y < x$   
 $\forall x \leq y. P \quad \equiv \quad \forall x. x \leq y \longrightarrow P$   
 $\exists x \leq y. P \quad \equiv \quad \exists x. x \leq y \wedge P$   
 Similarly for  $<$ ,  $\geq$  and  $>$   
 $LEAST x. P \quad \equiv \quad Least (\lambda x. P)$

## Lattices

Classes semilattice, lattice, distributive lattice and complete lattice (the latter in theory *Set*).

$inf \quad :: 'a \Rightarrow 'a \Rightarrow 'a$   
 $sup \quad :: 'a \Rightarrow 'a \Rightarrow 'a$   
 $Inf \quad :: 'a \text{ set} \Rightarrow 'a$   
 $Sup \quad :: 'a \text{ set} \Rightarrow 'a$

## Syntax

Available by loading theory *Lattice\_Syntax* in directory *Library*.

$x \sqsubseteq y \quad \equiv \quad x \leq y$   
 $x \sqsubset y \quad \equiv \quad x < y$   
 $x \sqcap y \quad \equiv \quad inf \ x \ y$   
 $x \sqcup y \quad \equiv \quad sup \ x \ y$   
 $\bigsqcap A \quad \equiv \quad Sup \ A$   
 $\bigsqcup A \quad \equiv \quad Inf \ A$   
 $\top \quad \quad \equiv \quad top$   
 $\perp \quad \quad \equiv \quad bot$

## Set

$\{\} \quad \quad \quad :: 'a \text{ set}$   
 $insert \quad :: 'a \Rightarrow 'a \text{ set} \Rightarrow 'a \text{ set}$   
 $Collect \quad :: ('a \Rightarrow bool) \Rightarrow 'a \text{ set}$   
 $op \in \quad \quad :: 'a \Rightarrow 'a \text{ set} \Rightarrow bool \quad \quad \quad (:)$   
 $op \cup \quad \quad :: 'a \text{ set} \Rightarrow 'a \text{ set} \Rightarrow 'a \text{ set} \quad \quad (Un)$   
 $op \cap \quad \quad :: 'a \text{ set} \Rightarrow 'a \text{ set} \Rightarrow 'a \text{ set} \quad \quad (Int)$

$UNION :: 'a \text{ set} \Rightarrow ('a \Rightarrow 'b \text{ set}) \Rightarrow 'b \text{ set}$   
 $INTER :: 'a \text{ set} \Rightarrow ('a \Rightarrow 'b \text{ set}) \Rightarrow 'b \text{ set}$   
 $Union :: 'a \text{ set set} \Rightarrow 'a \text{ set}$   
 $Inter :: 'a \text{ set set} \Rightarrow 'a \text{ set}$   
 $Pow :: 'a \text{ set} \Rightarrow 'a \text{ set set}$   
 $UNIV :: 'a \text{ set}$   
 $op \text{ ' } :: ('a \Rightarrow 'b) \Rightarrow 'a \text{ set} \Rightarrow 'b \text{ set}$   
 $Ball :: 'a \text{ set} \Rightarrow ('a \Rightarrow \text{bool}) \Rightarrow \text{bool}$   
 $Bex :: 'a \text{ set} \Rightarrow ('a \Rightarrow \text{bool}) \Rightarrow \text{bool}$

## Syntax

$\{a_1, \dots, a_n\} \equiv insert\ a_1\ (\dots\ (insert\ a_n\ \{\})\dots)$   
 $a \notin A \equiv \neg(x \in A)$   
 $A \subseteq B \equiv A \leq B$   
 $A \subset B \equiv A < B$   
 $A \supseteq B \equiv B \leq A$   
 $A \supset B \equiv B < A$   
 $\{x. P\} \equiv Collect\ (\lambda x. P)$   
 $\{t \mid x_1 \dots x_n. P\} \equiv \{v. \exists x_1 \dots x_n. v = t \wedge P\}$   
 $\bigcup_{x \in I}. A \equiv UNION\ I\ (\lambda x. A) \quad (\text{UN})$   
 $\bigcup x. A \equiv UNION\ UNIV\ (\lambda x. A)$   
 $\bigcap_{x \in I}. A \equiv INTER\ I\ (\lambda x. A) \quad (\text{INT})$   
 $\bigcap x. A \equiv INTER\ UNIV\ (\lambda x. A)$   
 $\forall x \in A. P \equiv Ball\ A\ (\lambda x. P)$   
 $\exists x \in A. P \equiv Bex\ A\ (\lambda x. P)$   
 $range\ f \equiv f \text{ ' } UNIV$

## Fun

$id :: 'a \Rightarrow 'a$   
 $op \circ :: ('a \Rightarrow 'b) \Rightarrow ('c \Rightarrow 'a) \Rightarrow 'c \Rightarrow 'b \quad (\text{o})$   
 $inj\_on :: ('a \Rightarrow 'b) \Rightarrow 'a \text{ set} \Rightarrow \text{bool}$   
 $inj :: ('a \Rightarrow 'b) \Rightarrow \text{bool}$   
 $surj :: ('a \Rightarrow 'b) \Rightarrow \text{bool}$   
 $bij :: ('a \Rightarrow 'b) \Rightarrow \text{bool}$   
 $bij\_betw :: ('a \Rightarrow 'b) \Rightarrow 'a \text{ set} \Rightarrow 'b \text{ set} \Rightarrow \text{bool}$   
 $fun\_upd :: ('a \Rightarrow 'b) \Rightarrow 'a \Rightarrow 'b \Rightarrow 'a \Rightarrow 'b$

## Syntax

$f(x := y) \equiv fun\_upd\ f\ x\ y$   
 $f(x_1 := y_1, \dots, x_n := y_n) \equiv f(x_1 := y_1) \dots (x_n := y_n)$

## Hilbert\_Choice

Hilbert's selection ( $\varepsilon$ ) operator: *SOME*  $x. P$ .

$inv\_into :: 'a\ set \Rightarrow ('a \Rightarrow 'b) \Rightarrow 'b \Rightarrow 'a$

### Syntax

$inv \equiv inv\_into\ UNIV$

## Fixed Points

Theory: *Inductive*.

Least and greatest fixed points in a complete lattice  $'a$ :

$lfp :: ('a \Rightarrow 'a) \Rightarrow 'a$

$gfp :: ('a \Rightarrow 'a) \Rightarrow 'a$

Note that in particular sets  $('a \Rightarrow bool)$  are complete lattices.

## Sum\_Type

Type constructor  $+$ .

$Inl :: 'a \Rightarrow 'a + 'b$

$Inr :: 'a \Rightarrow 'b + 'a$

$op\ <+> :: 'a\ set \Rightarrow 'b\ set \Rightarrow ('a + 'b)\ set$

## Product\_Type

Types *unit* and  $\times$ .

$() :: unit$

$Pair :: 'a \Rightarrow 'b \Rightarrow 'a \times 'b$

$fst :: 'a \times 'b \Rightarrow 'a$

$snd :: 'a \times 'b \Rightarrow 'b$

$split :: ('a \Rightarrow 'b \Rightarrow 'c) \Rightarrow 'a \times 'b \Rightarrow 'c$

$curry :: ('a \times 'b \Rightarrow 'c) \Rightarrow 'a \Rightarrow 'b \Rightarrow 'c$

$Sigma :: 'a\ set \Rightarrow ('a \Rightarrow 'b\ set) \Rightarrow ('a \times 'b)\ set$

### Syntax

$(a, b) \equiv Pair\ a\ b$

$\lambda(x, y). t \equiv split\ (\lambda x\ y. t)$

$A \times B \equiv Sigma\ A\ (\lambda_. B)$  ( $<*>$ )

Pairs may be nested. Nesting to the right is printed as a tuple, e.g.  $(a, b, c)$  is really  $(a, (b, c))$ . Pattern matching with pairs and tuples extends to all binders, e.g.

$\forall (x, y) \in A. P, \{(x, y). P\}$ , etc.

## Relation

*converse* :: ('a × 'b) set ⇒ ('b × 'a) set  
*op O* :: ('a × 'b) set ⇒ ('b × 'c) set ⇒ ('a × 'c) set  
*op “* :: ('a × 'b) set ⇒ 'a set ⇒ 'b set  
*inv\_image* :: ('a × 'a) set ⇒ ('b ⇒ 'a) ⇒ ('b × 'b) set  
*Id\_on* :: 'a set ⇒ ('a × 'a) set  
*Id* :: ('a × 'a) set  
*Domain* :: ('a × 'b) set ⇒ 'a set  
*Range* :: ('a × 'b) set ⇒ 'b set  
*Field* :: ('a × 'a) set ⇒ 'a set  
*refl\_on* :: 'a set ⇒ ('a × 'a) set ⇒ bool  
*refl* :: ('a × 'a) set ⇒ bool  
*sym* :: ('a × 'a) set ⇒ bool  
*antisym* :: ('a × 'a) set ⇒ bool  
*trans* :: ('a × 'a) set ⇒ bool  
*irrefl* :: ('a × 'a) set ⇒ bool  
*total\_on* :: 'a set ⇒ ('a × 'a) set ⇒ bool  
*total* :: ('a × 'a) set ⇒ bool

### Syntax

$r^{-1} \equiv \text{converse } r \quad (\hat{-}1)$

Type synonym  $'a \text{ rel} = ('a \times 'a) \text{ set}$

## Equiv\_Relations

*equiv* :: 'a set ⇒ ('a × 'a) set ⇒ bool  
*op //* :: 'a set ⇒ ('a × 'a) set ⇒ 'a set set  
*congruent* :: ('a × 'a) set ⇒ ('a ⇒ 'b) ⇒ bool  
*congruent2* :: ('a × 'a) set ⇒ ('b × 'b) set ⇒ ('a ⇒ 'b ⇒ 'c) ⇒ bool

### Syntax

$f \text{ respects } r \equiv \text{congruent } r \ f$   
 $f \text{ respects2 } r \equiv \text{congruent2 } r \ r \ f$

## Transitive\_Closure

$rtrancl :: ('a \times 'a) set \Rightarrow ('a \times 'a) set$   
 $trancl :: ('a \times 'a) set \Rightarrow ('a \times 'a) set$   
 $reflcl :: ('a \times 'a) set \Rightarrow ('a \times 'a) set$   
 $acyclic :: ('a \times 'a) set \Rightarrow bool$   
 $op ^^ :: ('a \times 'a) set \Rightarrow nat \Rightarrow ('a \times 'a) set$

### Syntax

$r^* \equiv rtrancl\ r \quad (\wedge^*)$   
 $r^+ \equiv trancl\ r \quad (\wedge^+)$   
 $r^- \equiv reflcl\ r \quad (\wedge=)$

## Algebra

Theories *Groups*, *Rings*, *Fields* and *Divides* define a large collection of classes describing common algebraic structures from semigroups up to fields. Everything is done in terms of overloaded operators:

$0 \quad \quad \quad :: 'a$   
 $1 \quad \quad \quad :: 'a$   
 $op + \quad :: 'a \Rightarrow 'a \Rightarrow 'a$   
 $op - \quad :: 'a \Rightarrow 'a \Rightarrow 'a$   
 $uminus :: 'a \Rightarrow 'a \quad \quad \quad (-)$   
 $op * \quad :: 'a \Rightarrow 'a \Rightarrow 'a$   
 $inverse :: 'a \Rightarrow 'a$   
 $op / \quad :: 'a \Rightarrow 'a \Rightarrow 'a$   
 $abs \quad \quad :: 'a \Rightarrow 'a$   
 $sgn \quad \quad :: 'a \Rightarrow 'a$   
 $op dvd \quad :: 'a \Rightarrow 'a \Rightarrow bool$   
 $op div \quad :: 'a \Rightarrow 'a \Rightarrow 'a$   
 $op mod \quad :: 'a \Rightarrow 'a \Rightarrow 'a$

### Syntax

$|x| \equiv abs\ x$

## Nat

**datatype**  $nat = 0 \mid Suc\ nat$

$op + \quad op - \quad op * \quad op ^ \quad op div \quad op mod \quad op dvd$   
 $op \leq \quad op < \quad min \quad max \quad Min \quad Max$

$of\_nat :: nat \Rightarrow 'a$   
 $op \wedge \wedge :: ('a \Rightarrow 'a) \Rightarrow nat \Rightarrow 'a \Rightarrow 'a$

## Int

Type *int*

$op + \quad op - \quad uminus \quad op * \quad op \wedge \quad op \text{ div} \quad op \text{ mod} \quad op \text{ dvd}$   
 $op \leq \quad op < \quad min \quad max \quad Min \quad Max$   
 $abs \quad sgn$

$nat :: int \Rightarrow nat$

$of\_int :: int \Rightarrow 'a$

$\mathbb{Z} :: 'a \text{ set} \quad (\text{Ints})$

### Syntax

$int \equiv of\_nat$

## Finite\_Set

$finite :: 'a \text{ set} \Rightarrow bool$

$card :: 'a \text{ set} \Rightarrow nat$

$Finite\_Set.fold :: ('a \Rightarrow 'b \Rightarrow 'b) \Rightarrow 'b \Rightarrow 'a \text{ set} \Rightarrow 'b$

$setsum :: ('a \Rightarrow 'b) \Rightarrow 'a \text{ set} \Rightarrow 'b$

$setprod :: ('a \Rightarrow 'b) \Rightarrow 'a \text{ set} \Rightarrow 'b$

### Syntax

$\sum A \quad \equiv \quad setsum (\lambda x. x) A \quad (\text{SUM})$

$\sum_{x \in A} t \quad \equiv \quad setsum (\lambda x. t) A$

$\sum_{x|P} t \quad \equiv \quad \sum x | P. t$

Similarly for  $\prod$  instead of  $\sum$  (PROD)

## Wellfounded

$wf :: ('a \times 'a) \text{ set} \Rightarrow bool$

$Wellfounded.acc :: ('a \times 'a) \text{ set} \Rightarrow 'a \text{ set}$

$measure :: ('a \Rightarrow nat) \Rightarrow ('a \times 'a) \text{ set}$

$op < *lex* > :: ('a \times 'a) \text{ set} \Rightarrow ('b \times 'b) \text{ set} \Rightarrow (('a \times 'b) \times 'a \times 'b) \text{ set}$

$op < *mlex* > :: ('a \Rightarrow nat) \Rightarrow ('a \times 'a) \text{ set} \Rightarrow ('a \times 'a) \text{ set}$

$less\_than :: (nat \times nat) \text{ set}$

$pred\_nat :: (nat \times nat) \text{ set}$

## Set \_Interval

*lessThan* :: 'a ⇒ 'a set  
*atMost* :: 'a ⇒ 'a set  
*greaterThan* :: 'a ⇒ 'a set  
*atLeast* :: 'a ⇒ 'a set  
*greaterThanLessThan* :: 'a ⇒ 'a ⇒ 'a set  
*atLeastLessThan* :: 'a ⇒ 'a ⇒ 'a set  
*greaterThanAtMost* :: 'a ⇒ 'a ⇒ 'a set  
*atLeastAtMost* :: 'a ⇒ 'a ⇒ 'a set

### Syntax

$\{..<y\}$  ≡ *lessThan* *y*  
 $\{..y\}$  ≡ *atMost* *y*  
 $\{x<..\}$  ≡ *greaterThan* *x*  
 $\{x..\}$  ≡ *atLeast* *x*  
 $\{x<..*y*\}$  ≡ *greaterThanLessThan* *x* *y*  
 $\{x..*y*\}$  ≡ *atLeastLessThan* *x* *y*  
 $\{x<..*y*\}$  ≡ *greaterThanAtMost* *x* *y*  
 $\{x..*y*\}$  ≡ *atLeastAtMost* *x* *y*  
 $\bigcup i \leq n. A$  ≡  $\bigcup i \in \{..n\}. A$   
 $\bigcup i < n. A$  ≡  $\bigcup i \in \{..<n\}. A$

Similarly for  $\bigcap$  instead of  $\bigcup$

$\sum x = a..b. t$  ≡ *setsum* ( $\lambda x. t$ )  $\{a..b\}$   
 $\sum x = a..*b*. t$  ≡ *setsum* ( $\lambda x. t$ )  $\{a..*b\}*$   
 $\sum x \leq b. t$  ≡ *setsum* ( $\lambda x. t$ )  $\{..b\}$   
 $\sum x < b. t$  ≡ *setsum* ( $\lambda x. t$ )  $\{..<b\}$

Similarly for  $\prod$  instead of  $\sum$

## Power

*op* ^ :: 'a ⇒ nat ⇒ 'a

## Option

**datatype** 'a option = None | Some 'a

*the* :: 'a option ⇒ 'a  
*map\_option* :: ('a ⇒ 'b) ⇒ 'a option ⇒ 'b option  
*set\_option* :: 'a option ⇒ 'a set  
*Option.bind* :: 'a option ⇒ ('a ⇒ 'b option) ⇒ 'b option



# List

**datatype** *'a list* = [] | *op* # *'a ('a list)*

*op* @ :: *'a list* ⇒ *'a list* ⇒ *'a list*  
*butlast* :: *'a list* ⇒ *'a list*  
*concat* :: *'a list list* ⇒ *'a list*  
*distinct* :: *'a list* ⇒ *bool*  
*drop* :: *nat* ⇒ *'a list* ⇒ *'a list*  
*dropWhile* :: (*'a* ⇒ *bool*) ⇒ *'a list* ⇒ *'a list*  
*filter* :: (*'a* ⇒ *bool*) ⇒ *'a list* ⇒ *'a list*  
*List.find* :: (*'a* ⇒ *bool*) ⇒ *'a list* ⇒ *'a option*  
*fold* :: (*'a* ⇒ *'b* ⇒ *'b*) ⇒ *'a list* ⇒ *'b* ⇒ *'b*  
*foldr* :: (*'a* ⇒ *'b* ⇒ *'b*) ⇒ *'a list* ⇒ *'b* ⇒ *'b*  
*foldl* :: (*'a* ⇒ *'b* ⇒ *'a*) ⇒ *'a* ⇒ *'b list* ⇒ *'a*  
*hd* :: *'a list* ⇒ *'a*  
*last* :: *'a list* ⇒ *'a*  
*length* :: *'a list* ⇒ *nat*  
*lenlex* :: (*'a* × *'a*) *set* ⇒ (*'a list* × *'a list*) *set*  
*lex* :: (*'a* × *'a*) *set* ⇒ (*'a list* × *'a list*) *set*  
*lexn* :: (*'a* × *'a*) *set* ⇒ *nat* ⇒ (*'a list* × *'a list*) *set*  
*lexord* :: (*'a* × *'a*) *set* ⇒ (*'a list* × *'a list*) *set*  
*listrel* :: (*'a* × *'b*) *set* ⇒ (*'a list* × *'b list*) *set*  
*listrel1* :: (*'a* × *'a*) *set* ⇒ (*'a list* × *'a list*) *set*  
*lists* :: *'a set* ⇒ *'a list set*  
*listset* :: *'a set list* ⇒ *'a list set*  
*listsum* :: *'a list* ⇒ *'a*  
*list\_all2* :: (*'a* ⇒ *'b* ⇒ *bool*) ⇒ *'a list* ⇒ *'b list* ⇒ *bool*  
*list\_update* :: *'a list* ⇒ *nat* ⇒ *'a* ⇒ *'a list*  
*map* :: (*'a* ⇒ *'b*) ⇒ *'a list* ⇒ *'b list*  
*measures* :: (*'a* ⇒ *nat*) *list* ⇒ (*'a* × *'a*) *set*  
*op !* :: *'a list* ⇒ *nat* ⇒ *'a*  
*remdups* :: *'a list* ⇒ *'a list*  
*removeAll* :: *'a* ⇒ *'a list* ⇒ *'a list*  
*remove1* :: *'a* ⇒ *'a list* ⇒ *'a list*  
*replicate* :: *nat* ⇒ *'a* ⇒ *'a list*  
*rev* :: *'a list* ⇒ *'a list*  
*rotate* :: *nat* ⇒ *'a list* ⇒ *'a list*  
*rotate1* :: *'a list* ⇒ *'a list*  
*set* :: *'a list* ⇒ *'a set*  
*sort* :: *'a list* ⇒ *'a list*  
*sorted* :: *'a list* ⇒ *bool*

$splice$       $:: 'a\ list \Rightarrow 'a\ list \Rightarrow 'a\ list$   
 $sublist$      $:: 'a\ list \Rightarrow nat\ set \Rightarrow 'a\ list$   
 $take$         $:: nat \Rightarrow 'a\ list \Rightarrow 'a\ list$   
 $takeWhile$   $:: ('a \Rightarrow bool) \Rightarrow 'a\ list \Rightarrow 'a\ list$   
 $tl$           $:: 'a\ list \Rightarrow 'a\ list$   
 $upt$          $:: nat \Rightarrow nat \Rightarrow nat\ list$   
 $upto$         $:: int \Rightarrow int \Rightarrow int\ list$   
 $zip$           $:: 'a\ list \Rightarrow 'b\ list \Rightarrow ('a \times 'b)\ list$

## Syntax

$[x_1, \dots, x_n]$     $\equiv x_1 \# \dots \# x_n \# []$   
 $[m..<n]$         $\equiv upt\ m\ n$   
 $[i..j]$           $\equiv upto\ i\ j$   
 $[e.\ x \leftarrow xs]$   $\equiv map\ (\lambda x.\ e)\ xs$   
 $[x \leftarrow xs.\ b]$   $\equiv filter\ (\lambda x.\ b)\ xs$   
 $xs[n := x]$       $\equiv list\_update\ xs\ n\ x$   
 $\sum x \leftarrow xs.\ e$   $\equiv listsum\ (map\ (\lambda x.\ e)\ xs)$

List comprehension:  $[e.\ q_1, \dots, q_n]$  where each qualifier  $q_i$  is either a generator  $pat \leftarrow e$  or a guard, i.e. boolean expression.

## Map

Maps model partial functions and are often used as finite tables. However, the domain of a map may be infinite.

$Map.empty$     $:: 'a \Rightarrow 'b\ option$   
 $op ++$         $:: ('a \Rightarrow 'b\ option) \Rightarrow ('a \Rightarrow 'b\ option) \Rightarrow 'a \Rightarrow 'b\ option$   
 $op \circ_m$       $:: ('a \Rightarrow 'b\ option) \Rightarrow ('c \Rightarrow 'a\ option) \Rightarrow 'c \Rightarrow 'b\ option$   
 $op |'$         $:: ('a \Rightarrow 'b\ option) \Rightarrow 'a\ set \Rightarrow 'a \Rightarrow 'b\ option$   
 $dom$          $:: ('a \Rightarrow 'b\ option) \Rightarrow 'a\ set$   
 $ran$          $:: ('a \Rightarrow 'b\ option) \Rightarrow 'b\ set$   
 $op \subseteq_m$      $:: ('a \Rightarrow 'b\ option) \Rightarrow ('a \Rightarrow 'b\ option) \Rightarrow bool$   
 $map\_of$       $:: ('a \times 'b)\ list \Rightarrow 'a \Rightarrow 'b\ option$   
 $map\_upds$     $:: ('a \Rightarrow 'b\ option) \Rightarrow 'a\ list \Rightarrow 'b\ list \Rightarrow 'a \Rightarrow 'b\ option$

## Syntax

$Map.empty$             $\equiv \lambda x.\ None$   
 $m(x \mapsto y)$         $\equiv m(x := Some\ y)$   
 $m(x_1 \mapsto y_1, \dots, x_n \mapsto y_n)$   $\equiv m(x_1 \mapsto y_1) \dots (x_n \mapsto y_n)$   
 $[x_1 \mapsto y_1, \dots, x_n \mapsto y_n]$   $\equiv Map.empty(x_1 \mapsto y_1, \dots, x_n \mapsto y_n)$   
 $m(xs \mapsto ys)$       $\equiv map\_upds\ m\ xs\ ys$

## Infix operators in Main

	Operator	precedence	associativity
Meta-logic	$\implies$	1	right
	$\equiv$	2	
Logic	$\wedge$	35	right
	$\vee$	30	right
	$\longrightarrow, \longleftrightarrow$	25	right
	$=, \neq$	50	left
Orderings	$\leq, <, \geq, >$	50	
Sets	$\subseteq, \subset, \supseteq, \supset$	50	
	$\in, \notin$	50	
	$\cap$	70	left
	$\cup$	65	left
Functions and Relations	$\circ$	55	left
	$'$	90	right
	$O$	75	right
	$''$	90	right
	$\wedge\wedge$	80	right
Numbers	$+, -$	65	left
	$*, /$	70	left
	$div, mod$	70	left
	$\wedge$	80	right
	$dvd$	50	
Lists	$\#, @$	65	right
	$!$	100	left