# 1 Equipollency

[readtex `set-theory/sections/02_functions/03_invertible-functions.ftl.tex`]

Let $u, v, w$ denote objects. Let $x, y, z$ denote sets. Let $f, g, h$ denote functions.

We conclude this part about functions by introducing the notion of *equipollency*: Two sets $x, y$ being equipollent expresses the idea of $x$ and $y$ having the same number of elements.

**Definition 1.1.** $x$ and $y$ are equipollent iff there exists a bijection between $x$ and $y$.

Let $x$ and $y$ are equipotent stand for $x$ and $y$ are equipollent.

**Proposition 1.2.** $x$ and $x$ are equipollent.

*Proof.* $\mathrm{id}_x$ is a bijection between $x$ and $x$. ☐

**Proposition 1.3.** If $x$ and $y$ are equipollent then $y$ and $x$ are equipollent.

*Proof.* Assume that $x$ and $y$ are equipollent. Take a bijection $f$ between $x$ and $y$. Then $f^{-1}$ is a bijection between $y$ and $x$. Hence $y$ and $x$ are equipollent. ☐

**Proposition 1.4.** If $x$ and $y$ are equipollent and $y$ and $z$ are equipollent then $x$ and $z$ are equipollent.

*Proof.* Assume that $x$ and $y$ are equipollent and $y$ and $z$ are equipollent. Take a bijection $f$ between $x$ and $y$. Take a bijection $g$ between $y$ and $z$. Then $g \circ f$ is a bijection between $x$ and $z$. Hence $x$ and $z$ are equipollent. ☐

**Proposition 1.5.** $x$ and $\emptyset$ are equipollent iff $x$ is empty.

*Proof.* Case $x$ and $\emptyset$ are equipollent. Take a bijection $f$ between $x$ and $\emptyset$. Assume that $x$ is nonempty. Take an element $u$ of $x$. Then $f(u) \in \emptyset$. Contradiction. End.

Case $x$ is empty. Then $x = \emptyset$. Hence $x$ and $\emptyset$ are equipollent. End. ☐