1 Ordering and addition

[readtex `arithmetic/sections/02_ordering/01_ordering.ftl.tex`]

Let k, l, m, n denote natural numbers.

In this section we will briefly study the behaviour of the ordering with respect to addition.

Proposition 1.1. We have

$$n < m \iff n + k < m + k.$$

Proof. Case $n < m$. Take a positive natural number l such that $m = n+l$. Then $m + k = (n + l) + k = (n + k) + l$. Hence $n + k < m + k$. End.

Case $n + k < m + k$. Take a positive natural number l such that $m + k = (n + k) + l$. $(n + k) + l = n + (k + l) = n + (l + k) = (n + l) + k$. Hence $m + k = (n + l) + k$. Thus $m = n + l$. Therefore $n < m$. End. ☐

Corollary 1.2. We have

$$n < m \iff k + n < k + m.$$

Proof. We have $k + n = n + k$ and $k + m = m + k$. Hence $k + n < k + m$ iff $n + k < m + k$. ☐

Corollary 1.3. $n \leq m$ iff $k + n \leq k + m$.

Corollary 1.4. $n \leq m$ iff $n + k \leq m + k$.