Russell's Paradox

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Russell's paradox is the assertion that every set (or class) theory that contains an unrestricted comprehension principle leads to contradictions. The following is a formalization of this fact expressed in Naproche's built-in language of *classes* and *objects*.

Theorem. (Russell) Assume that every class is an object. Then we have a contradiction.

Proof. Define

$$R = \{\, x \mid x \text{ is a class such that } x \notin x \,\}.$$

Then $R \in R$ iff $R \notin R$. Contradiction. □