

# Chapter 1

## Subtraction

File: arithmetic/sections/05\_subtraction.ftl.tex

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[readtex arithmetic/sections/04\_ordering.ftl.tex]

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**Definition 1.1.** Let  $n, m$  be natural numbers such that  $n \geq m$ .  $n - m$  is the natural number  $k$  such that  $n = m + k$ .

Let the difference of  $n$  and  $m$  stand for  $n - m$ .

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**Proposition 1.2.** Let  $n, m$  be natural numbers such that  $n \geq m$ . Then

$$n - m = 0 \quad \text{iff} \quad n = m.$$

*Proof.* Case  $n - m = 0$ . Then  $n = (n - m) + m = 0 + m = m$ . End.

Case  $n = m$ . We have  $(n - m) + m = n = m = 0 + m$ . Hence  $n - m = 0$ . End.  $\square$

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**Corollary 1.3.** Let  $n$  be a natural number. Then

$$n - n = 0.$$

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**Proposition 1.4.** Let  $n$  be a natural number. Then

$$n - 0 = n.$$

*Proof.* We have  $n = (n - 0) + 0 = n - 0$ . □

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**Proposition 1.5.** Let  $n, m$  be natural numbers such that  $n \geq m$ . Then

$$n - m \leq n.$$

*Proof.* We have  $(n - m) + m = n$ . Hence  $n - m \leq n$ . □

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**Proposition 1.6.** Let  $n, m$  be natural numbers such that  $n \geq m$ . Then

$$0 \neq m < n \text{ implies } n - m < n.$$

*Proof.* Assume  $0 \neq m < n$ . Suppose  $n - m \geq n$ . We have  $(n - m) + m = n$ . Then  $n + m = (n - m) + m = n = n + 0$ . Hence  $m = 0$ . Contradiction. □

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**Proposition 1.7.** Let  $n, m, k$  be natural numbers such that  $n \geq m$ . Then

$$(n - m) + k = (n + k) - m.$$

*Proof.* We have

$$\begin{aligned} & ((n - m) + k) + m \\ &= ((n - m) + m) + k \\ &= n + k \end{aligned}$$

$$= ((n + k) - m) + m.$$

Hence  $(n - m) + k = (n + k) - m$ . □

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**Corollary 1.8.** Let  $n, m, k$  be natural numbers such that  $n \geq m$ . Then

$$k + (n - m) = (k + n) - m.$$

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**Proposition 1.9.** Let  $n, m, k$  be natural numbers such that  $n \geq m + k$ . Then

$$(n - m) - k = n - (m + k).$$

*Proof.* We have

$$\begin{aligned} & ((n - m) - k) + (m + k) \\ &= (((n - m) - k) + k) + m \\ &= (n - m) + m \\ &= n \\ &= (n - (m + k)) + (m + k). \end{aligned}$$

Hence  $(n - m) - k = n - (m + k)$ . □