Cantor's theorem

In this document we prove Cantor's theorem:

Theorem. There is no surjection defined on a set M that surjects onto the powerset of M.

[synonym subset/-s] [synonym surject/-s]
Let M denote a set. Let f denote a function.

Axiom 1. M is setsized.

Axiom 2. Let x be an element of M. Then x is setsized.

Let the value of f at x stand for $f(x)$. Let f is defined on M stand for $\mathrm{dom}(f) = M$. Let the domain of f stand for $\mathrm{dom}(f)$.

Axiom 3. The value of f at any element of the domain of f is a set.

Definition 4. **(Subset)** A subset of M is a set N such that every element of N is an element of M.

Definition 5. The powerset of M is the class of subsets of M.

Axiom 6. The powerset of M is a set.

Definition 7. f surjects onto M iff every element of M is equal to the value of f at some element of the domain of f.

Theorem 8. **(Cantor)** No function that is defined on M surjects onto the powerset of M.

Proof. Proof by contradiction. Assume the contrary. Take a function f that is defined on M and surjects onto the powerset of M. Define $N = \{x \text{ in } M \mid x \notin f(x)\}$. Then for all sets x we have $x \in N$ if and only if $x \notin f(x) = N$. Contradiction. $\square$