What's in Main

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Abstract

This document lists the main types, functions and syntax provided by theory *Main*. It is meant as a quick overview of what is available. For infix operators and their precedences see the final section. The sophisticated class structure is only hinted at. For details see https://isabelle.in.tum.de/library/HOL.

HOL

The basic logic: $x = y$, *True*, *False*, $\neg\ P$, $P \wedge Q$, $P \vee Q$, $P \longrightarrow Q$, $\forall\, x.\ P$, $\exists\, x.\ P$, $\exists!x.\ P$, *THE* $x.\ P$.

undefined	::	$'a$
default	::	$'a$

Syntax

$x \neq y$	$\equiv$	$\neg\ (x = y)$	(~=)
$P \longleftrightarrow Q$	$\equiv$	$P = Q$	
if x *then* y *else* z	$\equiv$	*If* $x\ y\ z$	
let $x = e_1$ *in* e_2	$\equiv$	*Let* $e_1\ (\lambda x.\ e_2)$	

Orderings

A collection of classes defining basic orderings: preorder, partial order, linear order, dense linear order and wellorder.

$(\leq)$	$:: 'a \Rightarrow 'a \Rightarrow bool$	(<=)
$(<)$	$:: 'a \Rightarrow 'a \Rightarrow bool$	
Least	$:: ('a \Rightarrow bool) \Rightarrow 'a$	
Greatest	$:: ('a \Rightarrow bool) \Rightarrow 'a$	
min	$:: 'a \Rightarrow 'a \Rightarrow 'a$	
max	$:: 'a \Rightarrow 'a \Rightarrow 'a$	
top	$:: 'a$	
bot	$:: 'a$	
mono	$:: ('a \Rightarrow 'b) \Rightarrow bool$	
strict_mono	$:: ('a \Rightarrow 'b) \Rightarrow bool$	

Syntax

$x \geq y$	$\equiv$	$y \leq x$	(>=)
$x > y$	$\equiv$	$y < x$	
$\forall x{\leq}y.\ P$	$\equiv$	$\forall x.\ x \leq y \longrightarrow P$	
$\exists x{\leq}y.\ P$	$\equiv$	$\exists x.\ x \leq y \wedge P$	

Similarly for $<$, $\geq$ and $>$

$LEAST\ x.\ P$	$\equiv$	$Least\ (\lambda x.\ P)$
$GREATEST\ x.\ P$	$\equiv$	$Greatest\ (\lambda x.\ P)$

Lattices

Classes semilattice, lattice, distributive lattice and complete lattice (the latter in theory *HOL.Set*).

inf	$:: 'a \Rightarrow 'a \Rightarrow 'a$
sup	$:: 'a \Rightarrow 'a \Rightarrow 'a$
Inf	$:: 'a\ set \Rightarrow 'a$
Sup	$:: 'a\ set \Rightarrow 'a$

Syntax

Available by loading theory *Lattice_Syntax* in directory *Library*.

$x \sqsubseteq y$	$\equiv$	$x \leq y$
$x \sqsubset y$	$\equiv$	$x < y$
$x \sqcap y$	$\equiv$	$inf\ x\ y$
$x \sqcup y$	$\equiv$	$sup\ x\ y$
$\bigsqcap A$	$\equiv$	$Inf\ A$

$\bigsqcup A$	$\equiv$	$Sup\ A$
$\top$	$\equiv$	top
$\bot$	$\equiv$	bot

Set

$\{\}$	$:: 'a\ set$	
$insert$	$:: 'a \Rightarrow 'a\ set \Rightarrow 'a\ set$	
$Collect$	$:: ('a \Rightarrow bool) \Rightarrow 'a\ set$	
$(\in)$	$:: 'a \Rightarrow 'a\ set \Rightarrow bool$	(`:`)
$(\cup)$	$:: 'a\ set \Rightarrow 'a\ set \Rightarrow 'a\ set$	(`Un`)
$(\cap)$	$:: 'a\ set \Rightarrow 'a\ set \Rightarrow 'a\ set$	(`Int`)
$\bigcup$	$:: 'a\ set\ set \Rightarrow 'a\ set$	
$\bigcap$	$:: 'a\ set\ set \Rightarrow 'a\ set$	
Pow	$:: 'a\ set \Rightarrow 'a\ set\ set$	
$UNIV$	$:: 'a\ set$	
$(`)$	$:: ('a \Rightarrow 'b) \Rightarrow 'a\ set \Rightarrow 'b\ set$	
$Ball$	$:: 'a\ set \Rightarrow ('a \Rightarrow bool) \Rightarrow bool$	
Bex	$:: 'a\ set \Rightarrow ('a \Rightarrow bool) \Rightarrow bool$	

Syntax

$\{a_1,\ldots,a_n\}$	$\equiv$	$insert\ a_1\ (\ldots\ (insert\ a_n\ \{\})\ldots)$	
$a \notin A$	$\equiv$	$\neg(x \in A)$	
$A \subseteq B$	$\equiv$	$A \le B$	
$A \subset B$	$\equiv$	$A < B$	
$A \supseteq B$	$\equiv$	$B \le A$	
$A \supset B$	$\equiv$	$B < A$	
$\{x.\ P\}$	$\equiv$	$Collect\ (\lambda x.\ P)$	
$\{t \mid x_1\ \ldots\ x_n.\ P\}$	$\equiv$	$\{v.\ \exists x_1\ \ldots\ x_n.\ v = t \wedge P\}$	
$\bigcup x{\in}I.\ A$	$\equiv$	$\bigcup((\lambda x.\ A)\ `\ I)$	(`UN`)
$\bigcup x.\ A$	$\equiv$	$\bigcup((\lambda x.\ A)\ `\ UNIV)$	
$\bigcap x{\in}I.\ A$	$\equiv$	$\bigcap((\lambda x.\ A)\ `\ I)$	(`INT`)
$\bigcap x.\ A$	$\equiv$	$\bigcap((\lambda x.\ A)\ `\ UNIV)$	
$\forall x{\in}A.\ P$	$\equiv$	$Ball\ A\ (\lambda x.\ P)$	
$\exists x{\in}A.\ P$	$\equiv$	$Bex\ A\ (\lambda x.\ P)$	
$range\ f$	$\equiv$	$f\ `\ UNIV$	

Fun

id	$:: 'a \Rightarrow 'a$	
$(\circ)$	$:: ('a \Rightarrow 'b) \Rightarrow ('c \Rightarrow 'a) \Rightarrow 'c \Rightarrow 'b$	(o)
inj_on	$:: ('a \Rightarrow 'b) \Rightarrow 'a\ set \Rightarrow bool$	
inj	$:: ('a \Rightarrow 'b) \Rightarrow bool$	
surj	$:: ('a \Rightarrow 'b) \Rightarrow bool$	
bij	$:: ('a \Rightarrow 'b) \Rightarrow bool$	
bij_betw	$:: ('a \Rightarrow 'b) \Rightarrow 'a\ set \Rightarrow 'b\ set \Rightarrow bool$	
fun_upd	$:: ('a \Rightarrow 'b) \Rightarrow 'a \Rightarrow 'b \Rightarrow 'a \Rightarrow 'b$	

Syntax

$f(x := y)$	$\equiv$	*fun_upd f x y*
$f(x_1{:=}y_1,\ldots,x_n{:=}y_n)$	$\equiv$	$f(x_1{:=}y_1)\ldots(x_n{:=}y_n)$

Hilbert_Choice

Hilbert's selection (ε) operator: *SOME* $x.\ P$.

inv_into $:: 'a\ set \Rightarrow ('a \Rightarrow 'b) \Rightarrow 'b \Rightarrow 'a$

Syntax

inv $\equiv$ *inv_into UNIV*

Fixed Points

Theory: *HOL.Inductive*.

Least and greatest fixed points in a complete lattice $'a$:

lfp $:: ('a \Rightarrow 'a) \Rightarrow 'a$
gfp $:: ('a \Rightarrow 'a) \Rightarrow 'a$

Note that in particular sets ($'a \Rightarrow bool$) are complete lattices.

Sum_Type

Type constructor $+$.

Inl	$:: 'a \Rightarrow 'a + 'b$
Inr	$:: 'a \Rightarrow 'b + 'a$
$(<+>)$	$:: 'a\ set \Rightarrow 'b\ set \Rightarrow ('a + 'b)\ set$

Product_Type

Types *unit* and $\times$.

$()$	$:: unit$
$Pair$	$:: 'a \Rightarrow 'b \Rightarrow 'a \times 'b$
fst	$:: 'a \times 'b \Rightarrow 'a$
snd	$:: 'a \times 'b \Rightarrow 'b$
$case\_prod$	$:: ('a \Rightarrow 'b \Rightarrow 'c) \Rightarrow 'a \times 'b \Rightarrow 'c$
$curry$	$:: ('a \times 'b \Rightarrow 'c) \Rightarrow 'a \Rightarrow 'b \Rightarrow 'c$
$Sigma$	$:: 'a\ set \Rightarrow ('a \Rightarrow 'b\ set) \Rightarrow ('a \times 'b)\ set$

Syntax

$(a,\ b)$	$\equiv$	$Pair\ a\ b$
$\lambda(x,\ y).\ t$	$\equiv$	$case\_prod\ (\lambda x\ y.\ t)$
$A \times B$	$\equiv$	$Sigma\ A\ (\lambda\_.\ B)$

Pairs may be nested. Nesting to the right is printed as a tuple, e.g. $(a,\ b,\ c)$ is really $(a,\ (b,\ c))$. Pattern matching with pairs and tuples extends to all binders, e.g. $\forall (x,\ y)\in A.\ P$, $\{(x,\ y).\ P\}$, etc.

Relation

$converse$	$:: ('a \times 'b)\ set \Rightarrow ('b \times 'a)\ set$
(O)	$:: ('a \times 'b)\ set \Rightarrow ('b \times 'c)\ set \Rightarrow ('a \times 'c)\ set$
$(``)$	$:: ('a \times 'b)\ set \Rightarrow 'a\ set \Rightarrow 'b\ set$
inv_image	$:: ('a \times 'a)\ set \Rightarrow ('b \Rightarrow 'a) \Rightarrow ('b \times 'b)\ set$
Id_on	$:: 'a\ set \Rightarrow ('a \times 'a)\ set$
Id	$:: ('a \times 'a)\ set$
$Domain$	$:: ('a \times 'b)\ set \Rightarrow 'a\ set$
$Range$	$:: ('a \times 'b)\ set \Rightarrow 'b\ set$
$Field$	$:: ('a \times 'a)\ set \Rightarrow 'a\ set$
$refl\_on$	$:: 'a\ set \Rightarrow ('a \times 'a)\ set \Rightarrow bool$
$refl$	$:: ('a \times 'a)\ set \Rightarrow bool$
sym	$:: ('a \times 'a)\ set \Rightarrow bool$
$antisym$	$:: ('a \times 'a)\ set \Rightarrow bool$
$trans$	$:: ('a \times 'a)\ set \Rightarrow bool$
$irrefl$	$:: ('a \times 'a)\ set \Rightarrow bool$
$total\_on$	$:: 'a\ set \Rightarrow ('a \times 'a)\ set \Rightarrow bool$
$total$	$:: ('a \times 'a)\ set \Rightarrow bool$

Syntax

r^{-1}	$\equiv$	$converse\ r$	(`^-1`)

Type synonym $'a\ rel = ('a \times 'a)\ set$

Equiv_Relations

equiv	$:: 'a\ set \Rightarrow ('a \times 'a)\ set \Rightarrow bool$
(//)	$:: 'a\ set \Rightarrow ('a \times 'a)\ set \Rightarrow 'a\ set\ set$
congruent	$:: ('a \times 'a)\ set \Rightarrow ('a \Rightarrow 'b) \Rightarrow bool$
congruent2	$:: ('a \times 'a)\ set \Rightarrow ('b \times 'b)\ set \Rightarrow ('a \Rightarrow 'b \Rightarrow 'c) \Rightarrow bool$

Syntax

f respects r	$\equiv$	*congruent r f*
f respects2 r	$\equiv$	*congruent2 r r f*

Transitive_Closure

rtrancl	$:: ('a \times 'a)\ set \Rightarrow ('a \times 'a)\ set$
trancl	$:: ('a \times 'a)\ set \Rightarrow ('a \times 'a)\ set$
reflcl	$:: ('a \times 'a)\ set \Rightarrow ('a \times 'a)\ set$
acyclic	$:: ('a \times 'a)\ set \Rightarrow bool$
(^^)	$:: ('a \times 'a)\ set \Rightarrow nat \Rightarrow ('a \times 'a)\ set$

Syntax

r^*	$\equiv$	*rtrancl r*	(^*)
r^+	$\equiv$	*trancl r*	(^+)
$r^=$	$\equiv$	*reflcl r*	(^=)

Algebra

Theories *HOL.Groups*, *HOL.Rings*, *HOL.Fields* and *HOL.Divides* define a large collection of classes describing common algebraic structures from semi-groups up to fields. Everything is done in terms of overloaded operators:

0	$:: 'a$
1	$:: 'a$
(+)	$:: 'a \Rightarrow 'a \Rightarrow 'a$
(−)	$:: 'a \Rightarrow 'a \Rightarrow 'a$

$uminus$	$:: 'a \Rightarrow 'a$	(-)
$(*)$	$:: 'a \Rightarrow 'a \Rightarrow 'a$	
$inverse$	$:: 'a \Rightarrow 'a$	
(div)	$:: 'a \Rightarrow 'a \Rightarrow 'a$	
abs	$:: 'a \Rightarrow 'a$	
sgn	$:: 'a \Rightarrow 'a$	
(dvd)	$:: 'a \Rightarrow 'a \Rightarrow bool$	
(div)	$:: 'a \Rightarrow 'a \Rightarrow 'a$	
(mod)	$:: 'a \Rightarrow 'a \Rightarrow 'a$	

Syntax

$|x| \equiv abs\ x$

Nat

datatype $nat = 0 \mid Suc\ nat$

$(+)$	$(-)$	$(*)$	$(\hat{\ })$	(div)	(mod)	(dvd)
$(\leq)$	$(<)$	min	max	Min	Max	

of_nat	$:: nat \Rightarrow 'a$
$(\hat{\ }\hat{\ })$	$:: ('a \Rightarrow 'a) \Rightarrow nat \Rightarrow 'a \Rightarrow 'a$

Int

Type *int*

$(+)$	$(-)$	$uminus$	$(*)$	$(\hat{\ })$	(div)	(mod)	(dvd)
$(\leq)$	$(<)$	min	max	Min	Max		
abs	sgn						

nat	$:: int \Rightarrow nat$	
of_int	$:: int \Rightarrow 'a$	
$\mathbb{Z}$	$:: 'a\ set$	(`Ints`)

Syntax

$int \equiv of\_nat$

Finite_Set

finite	::	$'a\ set \Rightarrow bool$
card	::	$'a\ set \Rightarrow nat$
Finite_Set.fold	::	$('a \Rightarrow 'b \Rightarrow 'b) \Rightarrow 'b \Rightarrow 'a\ set \Rightarrow 'b$

Lattices_Big

Min	::	$'a\ set \Rightarrow 'a$
Max	::	$'a\ set \Rightarrow 'a$
arg_min	::	$('a \Rightarrow 'b) \Rightarrow ('a \Rightarrow bool) \Rightarrow 'a$
is_arg_min	::	$('a \Rightarrow 'b) \Rightarrow ('a \Rightarrow bool) \Rightarrow 'a \Rightarrow bool$
arg_max	::	$('a \Rightarrow 'b) \Rightarrow ('a \Rightarrow bool) \Rightarrow 'a$
is_arg_max	::	$('a \Rightarrow 'b) \Rightarrow ('a \Rightarrow bool) \Rightarrow 'a \Rightarrow bool$

Syntax

$ARG\_MIN\, f\, x.\ P$	$\equiv$	$arg\_min\, f\ (\lambda x.\ P)$
$ARG\_MAX\, f\, x.\ P$	$\equiv$	$arg\_max\, f\ (\lambda x.\ P)$

Groups_Big

sum	::	$('a \Rightarrow 'b) \Rightarrow 'a\ set \Rightarrow 'b$
prod	::	$('a \Rightarrow 'b) \Rightarrow 'a\ set \Rightarrow 'b$

Syntax

$\sum\ A$	$\equiv$	$sum\ (\lambda x.\ x)\ A$	(`SUM`)
$\sum x{\in}A.\ t$	$\equiv$	$sum\ (\lambda x.\ t)\ A$	
$\sum x{\mid}P.\ t$	$\equiv$	$\sum x\ \mid\ P.\ t$	
Similarly for $\prod$ instead of $\sum$			(`PROD`)

Wellfounded

wf	::	$('a \times 'a)\ set \Rightarrow bool$
Wellfounded.acc	::	$('a \times 'a)\ set \Rightarrow 'a\ set$
measure	::	$('a \Rightarrow nat) \Rightarrow ('a \times 'a)\ set$
$(<\!*lex*\!>)$	::	$('a \times 'a)\ set \Rightarrow ('b \times 'b)\ set \Rightarrow (('a \times 'b) \times 'a \times 'b)\ set$
$(<\!*mlex*\!>)$	::	$('a \Rightarrow nat) \Rightarrow ('a \times 'a)\ set \Rightarrow ('a \times 'a)\ set$

$less\_than :: (nat \times nat)\ set$
$pred\_nat :: (nat \times nat)\ set$

Set_Interval

$lessThan$	$:: 'a \Rightarrow 'a\ set$
$atMost$	$:: 'a \Rightarrow 'a\ set$
$greaterThan$	$:: 'a \Rightarrow 'a\ set$
$atLeast$	$:: 'a \Rightarrow 'a\ set$
$greaterThanLessThan$	$:: 'a \Rightarrow 'a \Rightarrow 'a\ set$
$atLeastLessThan$	$:: 'a \Rightarrow 'a \Rightarrow 'a\ set$
$greaterThanAtMost$	$:: 'a \Rightarrow 'a \Rightarrow 'a\ set$
$atLeastAtMost$	$:: 'a \Rightarrow 'a \Rightarrow 'a\ set$

Syntax

$\{..<y\}$	$\equiv$	$lessThan\ y$
$\{..y\}$	$\equiv$	$atMost\ y$
$\{x<..\}$	$\equiv$	$greaterThan\ x$
$\{x..\}$	$\equiv$	$atLeast\ x$
$\{x<..<y\}$	$\equiv$	$greaterThanLessThan\ x\ y$
$\{x..<y\}$	$\equiv$	$atLeastLessThan\ x\ y$
$\{x<..y\}$	$\equiv$	$greaterThanAtMost\ x\ y$
$\{x..y\}$	$\equiv$	$atLeastAtMost\ x\ y$
$\bigcup i\le n.\ A$	$\equiv$	$\bigcup i \in \{..n\}.\ A$
$\bigcup i<n.\ A$	$\equiv$	$\bigcup i \in \{..<n\}.\ A$
Similarly for $\bigcap$ instead of $\bigcup$		
$\sum x = a..b.\ t$	$\equiv$	$sum\ (\lambda x.\ t)\ \{a..b\}$
$\sum x = a..<b.\ t$	$\equiv$	$sum\ (\lambda x.\ t)\ \{a..<b\}$
$\sum x\le b.\ t$	$\equiv$	$sum\ (\lambda x.\ t)\ \{..b\}$
$\sum x<b.\ t$	$\equiv$	$sum\ (\lambda x.\ t)\ \{..<b\}$
Similarly for $\prod$ instead of $\sum$		

Power

$(\hat{\ }) :: 'a \Rightarrow nat \Rightarrow 'a$

Option

datatype $'a\ option = None \mid Some\ 'a$

the	$:: 'a\ option \Rightarrow 'a$
map_option	$:: ('a \Rightarrow 'b) \Rightarrow 'a\ option \Rightarrow 'b\ option$
set_option	$:: 'a\ option \Rightarrow 'a\ set$
$Option.bind$	$:: 'a\ option \Rightarrow ('a \Rightarrow 'b\ option) \Rightarrow 'b\ option$

List

datatype $'a\ list = [] \mid (\#)\ 'a\ ('a\ list)$

$(@)$	$:: 'a\ list \Rightarrow 'a\ list \Rightarrow 'a\ list$
$butlast$	$:: 'a\ list \Rightarrow 'a\ list$
$concat$	$:: 'a\ list\ list \Rightarrow 'a\ list$
$distinct$	$:: 'a\ list \Rightarrow bool$
$drop$	$:: nat \Rightarrow 'a\ list \Rightarrow 'a\ list$
$dropWhile$	$:: ('a \Rightarrow bool) \Rightarrow 'a\ list \Rightarrow 'a\ list$
$filter$	$:: ('a \Rightarrow bool) \Rightarrow 'a\ list \Rightarrow 'a\ list$
$find$	$:: ('a \Rightarrow bool) \Rightarrow 'a\ list \Rightarrow 'a\ option$
$fold$	$:: ('a \Rightarrow 'b \Rightarrow 'b) \Rightarrow 'a\ list \Rightarrow 'b \Rightarrow 'b$
$foldr$	$:: ('a \Rightarrow 'b \Rightarrow 'b) \Rightarrow 'a\ list \Rightarrow 'b \Rightarrow 'b$
$foldl$	$:: ('a \Rightarrow 'b \Rightarrow 'a) \Rightarrow 'a \Rightarrow 'b\ list \Rightarrow 'a$
hd	$:: 'a\ list \Rightarrow 'a$
$last$	$:: 'a\ list \Rightarrow 'a$
$length$	$:: 'a\ list \Rightarrow nat$
$lenlex$	$:: ('a \times 'a)\ set \Rightarrow ('a\ list \times 'a\ list)\ set$
lex	$:: ('a \times 'a)\ set \Rightarrow ('a\ list \times 'a\ list)\ set$
$lexn$	$:: ('a \times 'a)\ set \Rightarrow nat \Rightarrow ('a\ list \times 'a\ list)\ set$
$lexord$	$:: ('a \times 'a)\ set \Rightarrow ('a\ list \times 'a\ list)\ set$
$listrel$	$:: ('a \times 'b)\ set \Rightarrow ('a\ list \times 'b\ list)\ set$
$listrel1$	$:: ('a \times 'a)\ set \Rightarrow ('a\ list \times 'a\ list)\ set$
$lists$	$:: 'a\ set \Rightarrow 'a\ list\ set$
$listset$	$:: 'a\ set\ list \Rightarrow 'a\ list\ set$
sum_list	$:: 'a\ list \Rightarrow 'a$
$prod\_list$	$:: 'a\ list \Rightarrow 'a$

$list\_all2$	::	$('a \Rightarrow 'b \Rightarrow bool) \Rightarrow 'a\ list \Rightarrow 'b\ list \Rightarrow bool$
$list\_update$	::	$'a\ list \Rightarrow nat \Rightarrow 'a \Rightarrow 'a\ list$
map	::	$('a \Rightarrow 'b) \Rightarrow 'a\ list \Rightarrow 'b\ list$
$measures$	::	$('a \Rightarrow nat)\ list \Rightarrow ('a \times 'a)\ set$
$(!)$	::	$'a\ list \Rightarrow nat \Rightarrow 'a$
$nths$	::	$'a\ list \Rightarrow nat\ set \Rightarrow 'a\ list$
$remdups$	::	$'a\ list \Rightarrow 'a\ list$
$removeAll$	::	$'a \Rightarrow 'a\ list \Rightarrow 'a\ list$
$remove1$	::	$'a \Rightarrow 'a\ list \Rightarrow 'a\ list$
$replicate$	::	$nat \Rightarrow 'a \Rightarrow 'a\ list$
rev	::	$'a\ list \Rightarrow 'a\ list$
$rotate$	::	$nat \Rightarrow 'a\ list \Rightarrow 'a\ list$
$rotate1$	::	$'a\ list \Rightarrow 'a\ list$
set	::	$'a\ list \Rightarrow 'a\ set$
$shuffles$	::	$'a\ list \Rightarrow 'a\ list \Rightarrow 'a\ list\ set$
$sort$	::	$'a\ list \Rightarrow 'a\ list$
$sorted$	::	$'a\ list \Rightarrow bool$
$sorted\_wrt$	::	$('a \Rightarrow 'a \Rightarrow bool) \Rightarrow 'a\ list \Rightarrow bool$
$splice$	::	$'a\ list \Rightarrow 'a\ list \Rightarrow 'a\ list$
$take$	::	$nat \Rightarrow 'a\ list \Rightarrow 'a\ list$
$takeWhile$	::	$('a \Rightarrow bool) \Rightarrow 'a\ list \Rightarrow 'a\ list$
tl	::	$'a\ list \Rightarrow 'a\ list$
upt	::	$nat \Rightarrow nat \Rightarrow nat\ list$
$upto$	::	$int \Rightarrow int \Rightarrow int\ list$
zip	::	$'a\ list \Rightarrow 'b\ list \Rightarrow ('a \times 'b)\ list$

Syntax

$[x_1,\ldots,x_n]$	$\equiv$	$x_1\ \#\ \ldots\ \#\ x_n\ \#\ []$
$[m..<n]$	$\equiv$	$upt\ m\ n$
$[i..j]$	$\equiv$	$upto\ i\ j$
$xs[n := x]$	$\equiv$	$list\_update\ xs\ n\ x$
$\sum x{\leftarrow}xs.\ e$	$\equiv$	$listsum\ (map\ (\lambda x.\ e)\ xs)$

Filter input syntax $[pat \leftarrow e.\ b]$, where pat is a tuple pattern, which stands for $filter\ (\lambda pat.\ b)\ e$.

List comprehension input syntax: $[e.\ q_1,\ \ldots,\ q_n]$ where each qualifier q_i is either a generator $pat \leftarrow e$ or a guard, i.e. boolean expression.

Map

Maps model partial functions and are often used as finite tables. However, the domain of a map may be infinite.

$Map.empty$	$::$	$'a \Rightarrow 'b\ option$
$(++)$	$::$	$('a \Rightarrow 'b\ option) \Rightarrow ('a \Rightarrow 'b\ option) \Rightarrow 'a \Rightarrow 'b\ option$
$(\circ_m)$	$::$	$('a \Rightarrow 'b\ option) \Rightarrow ('c \Rightarrow 'a\ option) \Rightarrow 'c \Rightarrow 'b\ option$
$(\mid`)$	$::$	$('a \Rightarrow 'b\ option) \Rightarrow 'a\ set \Rightarrow 'a \Rightarrow 'b\ option$
dom	$::$	$('a \Rightarrow 'b\ option) \Rightarrow 'a\ set$
ran	$::$	$('a \Rightarrow 'b\ option) \Rightarrow 'b\ set$
$(\subseteq_m)$	$::$	$('a \Rightarrow 'b\ option) \Rightarrow ('a \Rightarrow 'b\ option) \Rightarrow bool$
map_of	$::$	$('a \times 'b)\ list \Rightarrow 'a \Rightarrow 'b\ option$
map_upds	$::$	$('a \Rightarrow 'b\ option) \Rightarrow 'a\ list \Rightarrow 'b\ list \Rightarrow 'a \Rightarrow 'b\ option$

Syntax

$Map.empty$	$\equiv$	$Map.empty$
$m(x \mapsto y)$	$\equiv$	$m(x{:=}Some\ y)$
$m(x_1{\mapsto}y_1,\ldots,x_n{\mapsto}y_n)$	$\equiv$	$m(x_1{\mapsto}y_1)\ldots(x_n{\mapsto}y_n)$
$[x_1{\mapsto}y_1,\ldots,x_n{\mapsto}y_n]$	$\equiv$	$Map.empty(x_1{\mapsto}y_1,\ldots,x_n{\mapsto}y_n)$
$m(xs\ [\mapsto]\ ys)$	$\equiv$	$map\_upds\ m\ xs\ ys$

Infix operators in Main

	Operator	precedence	associativity
Meta-logic	$\Longrightarrow$	1	right
	$\equiv$	2	
Logic	$\wedge$	35	right
	$\vee$	30	right
	$\longrightarrow$, $\longleftrightarrow$	25	right
	$=$, $\neq$	50	left
Orderings	$\leq$, $<$, $\geq$, $>$	50	
Sets	$\subseteq$, $\subset$, $\supseteq$, $\supset$	50	
	$\in$, $\notin$	50	
	$\cap$	70	left
	$\cup$	65	left
Functions and Relations	$\circ$	55	left
	`	90	right
	O	75	right
	``	90	right
	^^	80	right
Numbers	$+$, $-$	65	left
	$*$, $/$	70	left
	div, *mod*	70	left
	^	80	right
	dvd	50	
Lists	#, @	65	right
	!	100	left