What's in Main

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Abstract

This document lists the main types, functions and syntax provided by theory *Main*. It is meant as a quick overview of what is available. For infix operators and their precedences see the final section. The sophisticated class structure is only hinted at. For details see https://isabelle.in.tum.de/library/HOL/HOL.

HOL

The basic logic: $x = y$, *True*, *False*, $\neg P$, $P \wedge Q$, $P \vee Q$, $P \longrightarrow Q$, $\forall x.\ P$, $\exists x.\ P$, $\exists! x.\ P$, $THE\ x.\ P$.

$undefined :: 'a$
$default :: 'a$

Syntax

$x \neq y$	$\equiv$	$\neg (x = y)$	(~=)
$P \longleftrightarrow Q$	$\equiv$	$P = Q$	
$if\ x\ then\ y\ else\ z$	$\equiv$	$If\ x\ y\ z$	
$let\ x = e_1\ in\ e_2$	$\equiv$	$Let\ e_1\ (\lambda x.\ e_2)$	

Orderings

A collection of classes defining basic orderings: preorder, partial order, linear order, dense linear order and wellorder.

$(\le)$	$:: 'a \Rightarrow 'a \Rightarrow bool$	(<=)
$(<)$	$:: 'a \Rightarrow 'a \Rightarrow bool$	
Least	$:: ('a \Rightarrow bool) \Rightarrow 'a$	
Greatest	$:: ('a \Rightarrow bool) \Rightarrow 'a$	
min	$:: 'a \Rightarrow 'a \Rightarrow 'a$	
max	$:: 'a \Rightarrow 'a \Rightarrow 'a$	
top	$:: 'a$	
bot	$:: 'a$	
mono	$:: ('a \Rightarrow 'b) \Rightarrow bool$	
strict_mono	$:: ('a \Rightarrow 'b) \Rightarrow bool$	

Syntax

$x \ge y$	$\equiv$	$y \le x$	(>=)
$x > y$	$\equiv$	$y < x$	
$\forall x{\le}y.\ P$	$\equiv$	$\forall x.\ x \le y \longrightarrow P$	
$\exists x{\le}y.\ P$	$\equiv$	$\exists x.\ x \le y \wedge P$	
Similarly for $<$, $\ge$ and $>$			
LEAST $x.\ P$	$\equiv$	*Least* $(\lambda x.\ P)$	
GREATEST $x.\ P$	$\equiv$	*Greatest* $(\lambda x.\ P)$	

Lattices

Classes semilattice, lattice, distributive lattice and complete lattice (the latter in theory *HOL.Set*).

inf	$:: 'a \Rightarrow 'a \Rightarrow 'a$
sup	$:: 'a \Rightarrow 'a \Rightarrow 'a$
Inf	$:: 'a\ set \Rightarrow 'a$
Sup	$:: 'a\ set \Rightarrow 'a$

Syntax

Available via **unbundle** *lattice_syntax*.

$x \sqsubseteq y$	$\equiv$	$x \le y$
$x \sqsubset y$	$\equiv$	$x < y$
$x \sqcap y$	$\equiv$	*inf* x y
$x \sqcup y$	$\equiv$	*sup* x y
$\bigsqcap A$	$\equiv$	*Inf* A
$\bigsqcup A$	$\equiv$	*Sup* A

$\top$	$\equiv$	*top*
$\bot$	$\equiv$	*bot*

Set

$\{\}$	$:: 'a\ set$	
insert	$:: 'a \Rightarrow 'a\ set \Rightarrow 'a\ set$	
Collect	$:: ('a \Rightarrow bool) \Rightarrow 'a\ set$	
$(\in)$	$:: 'a \Rightarrow 'a\ set \Rightarrow bool$	(`:`)
$(\cup)$	$:: 'a\ set \Rightarrow 'a\ set \Rightarrow 'a\ set$	(`Un`)
$(\cap)$	$:: 'a\ set \Rightarrow 'a\ set \Rightarrow 'a\ set$	(`Int`)
$\bigcup$	$:: 'a\ set\ set \Rightarrow 'a\ set$	
$\bigcap$	$:: 'a\ set\ set \Rightarrow 'a\ set$	
Pow	$:: 'a\ set \Rightarrow 'a\ set\ set$	
UNIV	$:: 'a\ set$	
$(`)$	$:: ('a \Rightarrow 'b) \Rightarrow 'a\ set \Rightarrow 'b\ set$	
Ball	$:: 'a\ set \Rightarrow ('a \Rightarrow bool) \Rightarrow bool$	
Bex	$:: 'a\ set \Rightarrow ('a \Rightarrow bool) \Rightarrow bool$	

Syntax

$\{a_1,\ldots,a_n\}$	$\equiv$	$insert\ a_1\ (\ldots\ (insert\ a_n\ \{\})\ldots)$	
$a \notin A$	$\equiv$	$\neg(x \in A)$	
$A \subseteq B$	$\equiv$	$A \leq B$	
$A \subset B$	$\equiv$	$A < B$	
$A \supseteq B$	$\equiv$	$B \leq A$	
$A \supset B$	$\equiv$	$B < A$	
$\{x.\ P\}$	$\equiv$	$Collect\ (\lambda x.\ P)$	
$\{t \mid x_1\ \ldots\ x_n.\ P\}$	$\equiv$	$\{v.\ \exists\, x_1\ \ldots\ x_n.\ v = t \wedge P\}$	
$\bigcup x{\in}I.\ A$	$\equiv$	$\bigcup((\lambda x.\ A)\ `\ I)$	(`UN`)
$\bigcup x.\ A$	$\equiv$	$\bigcup((\lambda x.\ A)\ `\ UNIV)$	
$\bigcap x{\in}I.\ A$	$\equiv$	$\bigcap((\lambda x.\ A)\ `\ I)$	(`INT`)
$\bigcap x.\ A$	$\equiv$	$\bigcap((\lambda x.\ A)\ `\ UNIV)$	
$\forall\, x{\in}A.\ P$	$\equiv$	$Ball\ A\ (\lambda x.\ P)$	
$\exists\, x{\in}A.\ P$	$\equiv$	$Bex\ A\ (\lambda x.\ P)$	
$range\ f$	$\equiv$	$f\ `\ UNIV$	

Fun

id	$:: 'a \Rightarrow 'a$	
$(\circ)$	$:: ('a \Rightarrow 'b) \Rightarrow ('c \Rightarrow 'a) \Rightarrow 'c \Rightarrow 'b$	(o)
inj_on	$:: ('a \Rightarrow 'b) \Rightarrow 'a\ set \Rightarrow bool$	
inj	$:: ('a \Rightarrow 'b) \Rightarrow bool$	
surj	$:: ('a \Rightarrow 'b) \Rightarrow bool$	
bij	$:: ('a \Rightarrow 'b) \Rightarrow bool$	
bij_betw	$:: ('a \Rightarrow 'b) \Rightarrow 'a\ set \Rightarrow 'b\ set \Rightarrow bool$	
fun_upd	$:: ('a \Rightarrow 'b) \Rightarrow 'a \Rightarrow 'b \Rightarrow 'a \Rightarrow 'b$	

Syntax

$f(x := y)$	$\equiv$	*fun_upd f x y*
$f(x_1{:=}y_1,\ldots,x_n{:=}y_n)$	$\equiv$	$f(x_1{:=}y_1)\ldots(x_n{:=}y_n)$

Hilbert_Choice

Hilbert's selection (ε) operator: *SOME x. P*.

inv_into $:: 'a\ set \Rightarrow ('a \Rightarrow 'b) \Rightarrow 'b \Rightarrow 'a$

Syntax

inv $\equiv$ *inv_into UNIV*

Fixed Points

Theory: *HOL.Inductive*.

Least and greatest fixed points in a complete lattice $'a$:

lfp $:: ('a \Rightarrow 'a) \Rightarrow 'a$
gfp $:: ('a \Rightarrow 'a) \Rightarrow 'a$

Note that in particular sets ($'a \Rightarrow bool$) are complete lattices.

Sum_Type

Type constructor +.

Inl	::	$'a \Rightarrow 'a + 'b$
Inr	::	$'a \Rightarrow 'b + 'a$
$(<+>)$	::	$'a\ set \Rightarrow 'b\ set \Rightarrow ('a + 'b)\ set$

Product_Type

Types *unit* and ×.

$()$	::	$unit$
$Pair$	::	$'a \Rightarrow 'b \Rightarrow 'a \times 'b$
fst	::	$'a \times 'b \Rightarrow 'a$
snd	::	$'a \times 'b \Rightarrow 'b$
$case\_prod$	::	$('a \Rightarrow 'b \Rightarrow 'c) \Rightarrow 'a \times 'b \Rightarrow 'c$
$curry$	::	$('a \times 'b \Rightarrow 'c) \Rightarrow 'a \Rightarrow 'b \Rightarrow 'c$
$Sigma$	::	$'a\ set \Rightarrow ('a \Rightarrow 'b\ set) \Rightarrow ('a \times 'b)\ set$

Syntax

$(a,\ b)$	$\equiv$	$Pair\ a\ b$
$\lambda(x,\ y).\ t$	$\equiv$	$case\_prod\ (\lambda x\ y.\ t)$
$A \times B$	$\equiv$	$Sigma\ A\ (\lambda\_.\ B)$

Pairs may be nested. Nesting to the right is printed as a tuple, e.g. $(a,\ b,\ c)$ is really $(a,\ (b,\ c))$. Pattern matching with pairs and tuples extends to all binders, e.g. $\forall (x,\ y)\in A.\ P$, $\{(x,\ y).\ P\}$, etc.

Relation

$converse$	::	$('a \times 'b)\ set \Rightarrow ('b \times 'a)\ set$
(O)	::	$('a \times 'b)\ set \Rightarrow ('b \times 'c)\ set \Rightarrow ('a \times 'c)\ set$
$(``)$	::	$('a \times 'b)\ set \Rightarrow 'a\ set \Rightarrow 'b\ set$
inv_image	::	$('a \times 'a)\ set \Rightarrow ('b \Rightarrow 'a) \Rightarrow ('b \times 'b)\ set$
Id_on	::	$'a\ set \Rightarrow ('a \times 'a)\ set$
Id	::	$('a \times 'a)\ set$
$Domain$	::	$('a \times 'b)\ set \Rightarrow 'a\ set$
$Range$	::	$('a \times 'b)\ set \Rightarrow 'b\ set$
$Field$	::	$('a \times 'a)\ set \Rightarrow 'a\ set$
$refl\_on$	::	$'a\ set \Rightarrow ('a \times 'a)\ set \Rightarrow bool$
$refl$	::	$('a \times 'a)\ set \Rightarrow bool$
sym	::	$('a \times 'a)\ set \Rightarrow bool$

antisym	::	$('a \times 'a)\ set \Rightarrow bool$
trans	::	$('a \times 'a)\ set \Rightarrow bool$
irrefl	::	$('a \times 'a)\ set \Rightarrow bool$
total_on	::	$'a\ set \Rightarrow ('a \times 'a)\ set \Rightarrow bool$
total	::	$('a \times 'a)\ set \Rightarrow bool$

Syntax

$r^{-1} \quad \equiv \quad converse\ r \quad$ (`^-1`)

Type synonym $'a\ rel = ('a \times 'a)\ set$

Equiv_Relations

equiv	::	$'a\ set \Rightarrow ('a \times 'a)\ set \Rightarrow bool$
(//)	::	$'a\ set \Rightarrow ('a \times 'a)\ set \Rightarrow 'a\ set\ set$
congruent	::	$('a \times 'a)\ set \Rightarrow ('a \Rightarrow 'b) \Rightarrow bool$
congruent2	::	$('a \times 'a)\ set \Rightarrow ('b \times 'b)\ set \Rightarrow ('a \Rightarrow 'b \Rightarrow 'c) \Rightarrow bool$

Syntax

f respects r	$\equiv$	*congruent r f*
f respects2 r	$\equiv$	*congruent2 r r f*

Transitive_Closure

rtrancl	::	$('a \times 'a)\ set \Rightarrow ('a \times 'a)\ set$
trancl	::	$('a \times 'a)\ set \Rightarrow ('a \times 'a)\ set$
reflcl	::	$('a \times 'a)\ set \Rightarrow ('a \times 'a)\ set$
acyclic	::	$('a \times 'a)\ set \Rightarrow bool$
$(\widehat{\ \ })$	::	$('a \times 'a)\ set \Rightarrow nat \Rightarrow ('a \times 'a)\ set$

Syntax

r^*	$\equiv$	*rtrancl r*	(`^*`)
r^+	$\equiv$	*trancl r*	(`^+`)
$r^=$	$\equiv$	*reflcl r*	(`^=`)

Algebra

Theories *HOL.Groups*, *HOL.Rings*, *HOL.Fields* and *HOL.Divides* define a large collection of classes describing common algebraic structures from semigroups up to fields. Everything is done in terms of overloaded operators:

0	$:: 'a$	
1	$:: 'a$	
$(+)$	$:: 'a \Rightarrow 'a \Rightarrow 'a$	
$(-)$	$:: 'a \Rightarrow 'a \Rightarrow 'a$	
$uminus$	$:: 'a \Rightarrow 'a$	$(-)$
$(*)$	$:: 'a \Rightarrow 'a \Rightarrow 'a$	
$inverse$	$:: 'a \Rightarrow 'a$	
(div)	$:: 'a \Rightarrow 'a \Rightarrow 'a$	
abs	$:: 'a \Rightarrow 'a$	
sgn	$:: 'a \Rightarrow 'a$	
(dvd)	$:: 'a \Rightarrow 'a \Rightarrow bool$	
(div)	$:: 'a \Rightarrow 'a \Rightarrow 'a$	
(mod)	$:: 'a \Rightarrow 'a \Rightarrow 'a$	

Syntax

$|x| \quad \equiv \quad abs\ x$

Nat

datatype $nat = 0 \mid Suc\ nat$

$(+)$	$(-)$	$(*)$	$(\widehat{\ })$	(div)	(mod)	(dvd)
$(\le)$	$(<)$	min	max	Min	Max	

$of\_nat :: nat \Rightarrow 'a$
$(\widehat{\ }\widehat{\ }) :: ('a \Rightarrow 'a) \Rightarrow nat \Rightarrow 'a \Rightarrow 'a$

Int

Type *int*

$(+)$	$(-)$	$uminus$	$(*)$	$(\widehat{\ })$	(div)	(mod)	(dvd)
$(\le)$	$(<)$	min	max	Min	Max		
abs	sgn						

nat	$::\ int \Rightarrow nat$	
of_int	$::\ int \Rightarrow {}'a$	
$\mathbb{Z}$	$::\ {}'a\ set$	(Ints)

Syntax

$int \quad \equiv \quad of\_nat$

Finite_Set

$finite$	$::\ {}'a\ set \Rightarrow bool$
$card$	$::\ {}'a\ set \Rightarrow nat$
$Finite\_Set.fold$	$::\ ({}'a \Rightarrow {}'b \Rightarrow {}'b) \Rightarrow {}'b \Rightarrow {}'a\ set \Rightarrow {}'b$

Lattices_Big

Min	$::\ {}'a\ set \Rightarrow {}'a$
Max	$::\ {}'a\ set \Rightarrow {}'a$
arg_min	$::\ ({}'a \Rightarrow {}'b) \Rightarrow ({}'a \Rightarrow bool) \Rightarrow {}'a$
is_arg_min	$::\ ({}'a \Rightarrow {}'b) \Rightarrow ({}'a \Rightarrow bool) \Rightarrow {}'a \Rightarrow bool$
arg_max	$::\ ({}'a \Rightarrow {}'b) \Rightarrow ({}'a \Rightarrow bool) \Rightarrow {}'a$
is_arg_max	$::\ ({}'a \Rightarrow {}'b) \Rightarrow ({}'a \Rightarrow bool) \Rightarrow {}'a \Rightarrow bool$

Syntax

$ARG\_MIN\ f\ x.\ P$	$\equiv$	$arg\_min\ f\ (\lambda x.\ P)$
$ARG\_MAX\ f\ x.\ P$	$\equiv$	$arg\_max\ f\ (\lambda x.\ P)$

Groups_Big

$sum\ ::\ ({}'a \Rightarrow {}'b) \Rightarrow {}'a\ set \Rightarrow {}'b$
$prod\ ::\ ({}'a \Rightarrow {}'b) \Rightarrow {}'a\ set \Rightarrow {}'b$

Syntax

$\sum A$	$\equiv$	$sum\ (\lambda x.\ x)\ A$	(SUM)
$\sum x{\in}A.\ t$	$\equiv$	$sum\ (\lambda x.\ t)\ A$	
$\sum x\vert P.\ t$	$\equiv$	$\sum x \mid P.\ t$	

Similarly for $\prod$ instead of $\sum$ (PROD)

Wellfounded

wf	$:: ('a \times 'a)\ set \Rightarrow bool$
$Wellfounded.acc$	$:: ('a \times 'a)\ set \Rightarrow 'a\ set$
$measure$	$:: ('a \Rightarrow nat) \Rightarrow ('a \times 'a)\ set$
$(<*lex*>)$	$:: ('a \times 'a)\ set \Rightarrow ('b \times 'b)\ set \Rightarrow (('a \times 'b) \times 'a \times 'b)\ set$
$(<*mlex*>)$	$:: ('a \Rightarrow nat) \Rightarrow ('a \times 'a)\ set \Rightarrow ('a \times 'a)\ set$
$less\_than$	$:: (nat \times nat)\ set$
$pred\_nat$	$:: (nat \times nat)\ set$

Set_Interval

$lessThan$	$:: 'a \Rightarrow 'a\ set$
$atMost$	$:: 'a \Rightarrow 'a\ set$
$greaterThan$	$:: 'a \Rightarrow 'a\ set$
$atLeast$	$:: 'a \Rightarrow 'a\ set$
$greaterThanLessThan$	$:: 'a \Rightarrow 'a \Rightarrow 'a\ set$
$atLeastLessThan$	$:: 'a \Rightarrow 'a \Rightarrow 'a\ set$
$greaterThanAtMost$	$:: 'a \Rightarrow 'a \Rightarrow 'a\ set$
$atLeastAtMost$	$:: 'a \Rightarrow 'a \Rightarrow 'a\ set$

Syntax

$\{..<y\}$	$\equiv$	$lessThan\ y$
$\{..y\}$	$\equiv$	$atMost\ y$
$\{x<..\}$	$\equiv$	$greaterThan\ x$
$\{x..\}$	$\equiv$	$atLeast\ x$
$\{x<..<y\}$	$\equiv$	$greaterThanLessThan\ x\ y$
$\{x..<y\}$	$\equiv$	$atLeastLessThan\ x\ y$
$\{x<..y\}$	$\equiv$	$greaterThanAtMost\ x\ y$
$\{x..y\}$	$\equiv$	$atLeastAtMost\ x\ y$
$\bigcup i{\le}n.\ A$	$\equiv$	$\bigcup i \in \{..n\}.\ A$
$\bigcup i{<}n.\ A$	$\equiv$	$\bigcup i \in \{..<n\}.\ A$

Similarly for $\bigcap$ instead of $\bigcup$

$\sum x = a..b.\ t$	$\equiv$	$sum\ (\lambda x.\ t)\ \{a..b\}$
$\sum x = a..<b.\ t$	$\equiv$	$sum\ (\lambda x.\ t)\ \{a..<b\}$
$\sum x{\le}b.\ t$	$\equiv$	$sum\ (\lambda x.\ t)\ \{..b\}$
$\sum x{<}b.\ t$	$\equiv$	$sum\ (\lambda x.\ t)\ \{..<b\}$

Similarly for $\prod$ instead of $\sum$

Power

$(\hat{}) :: 'a \Rightarrow nat \Rightarrow 'a$

Option

datatype $'a\ option = None \mid Some\ 'a$

the	$:: 'a\ option \Rightarrow 'a$
map_option	$:: ('a \Rightarrow 'b) \Rightarrow 'a\ option \Rightarrow 'b\ option$
set_option	$:: 'a\ option \Rightarrow 'a\ set$
Option.bind	$:: 'a\ option \Rightarrow ('a \Rightarrow 'b\ option) \Rightarrow 'b\ option$

List

datatype $'a\ list = [] \mid (\#)\ 'a\ ('a\ list)$

(@)	$:: 'a\ list \Rightarrow 'a\ list \Rightarrow 'a\ list$
butlast	$:: 'a\ list \Rightarrow 'a\ list$
concat	$:: 'a\ list\ list \Rightarrow 'a\ list$
distinct	$:: 'a\ list \Rightarrow bool$
drop	$:: nat \Rightarrow 'a\ list \Rightarrow 'a\ list$
dropWhile	$:: ('a \Rightarrow bool) \Rightarrow 'a\ list \Rightarrow 'a\ list$
filter	$:: ('a \Rightarrow bool) \Rightarrow 'a\ list \Rightarrow 'a\ list$
find	$:: ('a \Rightarrow bool) \Rightarrow 'a\ list \Rightarrow 'a\ option$
fold	$:: ('a \Rightarrow 'b \Rightarrow 'b) \Rightarrow 'a\ list \Rightarrow 'b \Rightarrow 'b$
foldr	$:: ('a \Rightarrow 'b \Rightarrow 'b) \Rightarrow 'a\ list \Rightarrow 'b \Rightarrow 'b$
foldl	$:: ('a \Rightarrow 'b \Rightarrow 'a) \Rightarrow 'a \Rightarrow 'b\ list \Rightarrow 'a$
hd	$:: 'a\ list \Rightarrow 'a$
last	$:: 'a\ list \Rightarrow 'a$
length	$:: 'a\ list \Rightarrow nat$
lenlex	$:: ('a \times 'a)\ set \Rightarrow ('a\ list \times 'a\ list)\ set$
lex	$:: ('a \times 'a)\ set \Rightarrow ('a\ list \times 'a\ list)\ set$
lexn	$:: ('a \times 'a)\ set \Rightarrow nat \Rightarrow ('a\ list \times 'a\ list)\ set$
lexord	$:: ('a \times 'a)\ set \Rightarrow ('a\ list \times 'a\ list)\ set$
listrel	$:: ('a \times 'b)\ set \Rightarrow ('a\ list \times 'b\ list)\ set$
listrel1	$:: ('a \times 'a)\ set \Rightarrow ('a\ list \times 'a\ list)\ set$
lists	$:: 'a\ set \Rightarrow 'a\ list\ set$

listset	::	$'a\ set\ list \Rightarrow 'a\ list\ set$
sum_list	::	$'a\ list \Rightarrow 'a$
prod_list	::	$'a\ list \Rightarrow 'a$
list_all2	::	$('a \Rightarrow 'b \Rightarrow bool) \Rightarrow 'a\ list \Rightarrow 'b\ list \Rightarrow bool$
list_update	::	$'a\ list \Rightarrow nat \Rightarrow 'a \Rightarrow 'a\ list$
map	::	$('a \Rightarrow 'b) \Rightarrow 'a\ list \Rightarrow 'b\ list$
measures	::	$('a \Rightarrow nat)\ list \Rightarrow ('a \times 'a)\ set$
(!)	::	$'a\ list \Rightarrow nat \Rightarrow 'a$
nths	::	$'a\ list \Rightarrow nat\ set \Rightarrow 'a\ list$
remdups	::	$'a\ list \Rightarrow 'a\ list$
removeAll	::	$'a \Rightarrow 'a\ list \Rightarrow 'a\ list$
remove1	::	$'a \Rightarrow 'a\ list \Rightarrow 'a\ list$
replicate	::	$nat \Rightarrow 'a \Rightarrow 'a\ list$
rev	::	$'a\ list \Rightarrow 'a\ list$
rotate	::	$nat \Rightarrow 'a\ list \Rightarrow 'a\ list$
rotate1	::	$'a\ list \Rightarrow 'a\ list$
set	::	$'a\ list \Rightarrow 'a\ set$
shuffles	::	$'a\ list \Rightarrow 'a\ list \Rightarrow 'a\ list\ set$
sort	::	$'a\ list \Rightarrow 'a\ list$
sorted	::	$'a\ list \Rightarrow bool$
sorted_wrt	::	$('a \Rightarrow 'a \Rightarrow bool) \Rightarrow 'a\ list \Rightarrow bool$
splice	::	$'a\ list \Rightarrow 'a\ list \Rightarrow 'a\ list$
take	::	$nat \Rightarrow 'a\ list \Rightarrow 'a\ list$
takeWhile	::	$('a \Rightarrow bool) \Rightarrow 'a\ list \Rightarrow 'a\ list$
tl	::	$'a\ list \Rightarrow 'a\ list$
upt	::	$nat \Rightarrow nat \Rightarrow nat\ list$
upto	::	$int \Rightarrow int \Rightarrow int\ list$
zip	::	$'a\ list \Rightarrow 'b\ list \Rightarrow ('a \times 'b)\ list$

Syntax

$[x_1,\ldots,x_n]$	$\equiv$	$x_1\ \#\ \ldots\ \#\ x_n\ \#\ []$
$[m..<n]$	$\equiv$	$upt\ m\ n$
$[i..j]$	$\equiv$	$upto\ i\ j$
$xs[n := x]$	$\equiv$	$list\_update\ xs\ n\ x$
$\sum x{\leftarrow}xs.\ e$	$\equiv$	$listsum\ (map\ (\lambda x.\ e)\ xs)$

Filter input syntax $[pat \leftarrow e.\ b]$, where *pat* is a tuple pattern, which stands for $filter\ (\lambda pat.\ b)\ e$.

List comprehension input syntax: $[e.\ q_1,\ \ldots,\ q_n]$ where each qualifier q_i is either a generator $pat \leftarrow e$ or a guard, i.e. boolean expression.

Map

Maps model partial functions and are often used as finite tables. However, the domain of a map may be infinite.

Map.empty	$:: 'a \Rightarrow 'b\ option$
$(++)$	$:: ('a \Rightarrow 'b\ option) \Rightarrow ('a \Rightarrow 'b\ option) \Rightarrow 'a \Rightarrow 'b\ option$
$(\circ_m)$	$:: ('a \Rightarrow 'b\ option) \Rightarrow ('c \Rightarrow 'a\ option) \Rightarrow 'c \Rightarrow 'b\ option$
$(\|`)$	$:: ('a \Rightarrow 'b\ option) \Rightarrow 'a\ set \Rightarrow 'a \Rightarrow 'b\ option$
dom	$:: ('a \Rightarrow 'b\ option) \Rightarrow 'a\ set$
ran	$:: ('a \Rightarrow 'b\ option) \Rightarrow 'b\ set$
$(\subseteq_m)$	$:: ('a \Rightarrow 'b\ option) \Rightarrow ('a \Rightarrow 'b\ option) \Rightarrow bool$
map_of	$:: ('a \times 'b)\ list \Rightarrow 'a \Rightarrow 'b\ option$
map_upds	$:: ('a \Rightarrow 'b\ option) \Rightarrow 'a\ list \Rightarrow 'b\ list \Rightarrow 'a \Rightarrow 'b\ option$

Syntax

Map.empty	$\equiv$	$\lambda\_.\ None$
$m(x \mapsto y)$	$\equiv$	$m(x{:=}Some\ y)$
$m(x_1{\mapsto}y_1,\ldots,x_n{\mapsto}y_n)$	$\equiv$	$m(x_1{\mapsto}y_1)\ldots(x_n{\mapsto}y_n)$
$[x_1{\mapsto}y_1,\ldots,x_n{\mapsto}y_n]$	$\equiv$	$Map.empty(x_1{\mapsto}y_1,\ldots,x_n{\mapsto}y_n)$
$m(xs\ [\mapsto]\ ys)$	$\equiv$	*map_upds m xs ys*

Infix operators in Main

	Operator	precedence	associativity
Meta-logic	$\Longrightarrow$	1	right
	$\equiv$	2	
Logic	$\wedge$	35	right
	$\vee$	30	right
	$\longrightarrow$, $\longleftrightarrow$	25	right
	$=$, $\neq$	50	left
Orderings	$\leq$, $<$, $\geq$, $>$	50	
Sets	$\subseteq$, $\subset$, $\supseteq$, $\supset$	50	
	$\in$, $\notin$	50	
	$\cap$	70	left
	$\cup$	65	left
Functions and Relations	$\circ$	55	left
	`	90	right
	O	75	right
	``	90	right
	^^	80	right
Numbers	$+$, $-$	65	left
	$*$, $/$	70	left
	div, *mod*	70	left
	^	80	right
	dvd	50	
Lists	#, @	65	right
	!	100	left