

# Isabelle/HOL Exercises

## Advanced

### Merge Sort

#### Sorting with lists

For simplicity we sort natural numbers.

Define a predicate *sorted* that checks if each element in the list is less or equal to the following ones; *le n xs* should be true iff *n* is less or equal to all elements of *xs*.

```
consts
  le      :: "nat  $\Rightarrow$  nat list  $\Rightarrow$  bool"
  sorted :: "nat list  $\Rightarrow$  bool"
```

Define a function *count xs x* that counts how often *x* occurs in *xs*.

```
consts
  count :: "nat list  $\Rightarrow$  nat  $\Rightarrow$  nat"
```

#### Merge sort

Implement *merge sort*: a list is sorted by splitting it into two lists, sorting them separately, and merging the results.

With the help of *recdef* define two functions

```
consts merge :: "nat list  $\times$  nat list  $\Rightarrow$  nat list"
  msort  :: "nat list  $\Rightarrow$  nat list"
```

and show

```
theorem "sorted (msort xs)"
theorem "count (msort xs) x = count xs x"
```

You may have to prove lemmas about *ex.sorted* and *count*.

Hints:

- For *recdef* see Section 3.5 of the Isabelle/HOL tutorial.
- To split a list into two halves of almost equal length you can use the functions *n div 2*, *take* und *drop*, where *take n xs* returns the first *n* elements of *xs* and *drop n xs* the remainder.

- Here are some potentially useful lemmas:

*linorder\_not\_le*:  $(\neg x \leq y) = (y < x)$

*order\_less\_le*:  $(x < y) = (x \leq y \wedge x \neq y)$

*min\_def*:  $\min a b = (\text{if } a \leq b \text{ then } a \text{ else } b)$