

1 Jinja VCG

theory *JBC-VCG* = *JBC-SafetyLogic* + *VCG-Upgrades*:

1.1 Control Flow Graph

consts *xcpt-cond*::*jdbc-prog* \Rightarrow *cname* \Rightarrow *pos* \Rightarrow *expr*

defs *xcpt-cond-def*:

xcpt-cond Π *X p* \equiv (*case* (*cmd* Π *p*) *of* *None* \Rightarrow *TT* | *Some* *c* \Rightarrow (*case* *c*
of *New* *C* \Rightarrow *Eq* (*NewA* 0) (*Cn* *Null*)
| *Getfield* *F C* \Rightarrow *Eq* (*St* 0) (*Cn* *Null*)
| *Putfield* *F C* \Rightarrow *Eq* (*St* 1) (*Cn* *Null*)
| *Checkcast* *C* \Rightarrow *And* ((*Neg* (*Eq* (*St* 0) (*Cn* *Null*)))#(*map* (λ *Cl*. *Neg* (*Ty* (*St* 0) (*Class* *Cl*))) [*Cl* \in
(*classnames* (*fst* Π)). (*fst* Π) \vdash *Cl* \preceq^* *C*]))
| *Invoke* *M n* \Rightarrow *Eq* (*St* *n*) (*Cn* *Null*)
| *Throw* \Rightarrow (*if* *X* = *NullPointer* *then* *Neg* (*And* [*Neg* (*Eq* (*St* 0) (*Cn* *Null*)), *Neg* (*Ty* (*St* 0) (*Class* *X*))])
else *Ty* (*St* 0) (*Class* *X*))
| - \Rightarrow *TT*))

consts

succsInvoke::(*jdbc-prog* \times *mname* \times *nat* \times *pos*) \Rightarrow (*pos* \times *expr*) *list*

recdef *succsInvoke* {}

succsInvoke (Π, M, n, p) = (*case* *anF* Π *p* *of* *None* \Rightarrow []
| *Some* *A* \Rightarrow *concat* (*map* (λ *tp*. (*case* *tp* *of* *Void* \Rightarrow []
| *Boolean* \Rightarrow []
| *Integer* \Rightarrow []
| *NT* \Rightarrow []
| *Class* *X* \Rightarrow [(*fst* (*method* (*fst* Π) *X M*), *M*, 0), *And* [*Neg* (*xcpt-cond* Π *NullPointer* *p*), *Ty* (*St* *n*) (*Class* *X*)]])) (*extractTy* (*A*, *St* *n*))))

constdefs

succsNormal::*jdbc-prog* \Rightarrow *pos* \Rightarrow (*pos* \times *expr*) *list*

succsNormal Π *p* \equiv (*case* *cmd* Π *p* *of* *None* \Rightarrow []

| *Some* *c* \Rightarrow (*case* *c*

of *Load* *n* \Rightarrow [(*incA* *p*, *TT*)]

| *Store* *n* \Rightarrow [(*incA* *p*, *TT*)]

| *Push* *v* \Rightarrow [(*incA* *p*, *TT*)]

| *New* *C* \Rightarrow [(*incA* *p*, *Neg* (*xcpt-cond* Π *OutOfMemory* *p*))]

| *Getfield* *F C* \Rightarrow [(*incA* *p*, *Neg* (*xcpt-cond* Π *NullPointer* *p*))]

| *Putfield* *F C* \Rightarrow [(*incA* *p*, *Neg* (*xcpt-cond* Π *NullPointer* *p*))]

| *Checkcast* *C* \Rightarrow [(*incA* *p*, *Neg* (*xcpt-cond* Π *ClassCast* *p*))]

| *Invoke* *M n* \Rightarrow *succsInvoke* (Π, M, n, p)

| *Return* \Rightarrow *map* (λ *p'*. (*incA* *p'*, *Call* (*And* [*aF* Π *p'*, *Pos* *p'*])) (*callers* Π *p*))

| *Pop* \Rightarrow [(*incA* *p*, *TT*)]

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| IBin no  $\Rightarrow$  [(incA p, TT)]
| Goto t  $\Rightarrow$  let (C, M, n) = p in [((C, M, nat ((int n) + t)), TT)]
| CmpEq  $\Rightarrow$  [(incA p, TT)]
| IfIntCmp ro t  $\Rightarrow$  let (C, M, n) = p
    in [((C, M, nat ((int n) + t)), Rel (St 1) ro (St 0)),
        (incA p, Neg (Rel (St 1) ro (St 0)))]
| IfFalse t  $\Rightarrow$  let (C, M, n) = p
    in [((C, M, nat ((int n) + t)), Eq (St 0) (Cn (Bool False))),
        (incA p, Neg (Eq (St 0) (Cn (Bool False))))]
| Throw  $\Rightarrow$  []
))

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consts

match-ex-table-e :: 'm prog \Rightarrow cname \Rightarrow pc \Rightarrow ex-table \Rightarrow ex-entry option

primrec

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match-ex-table-e P C pc [] = None
match-ex-table-e P C pc (e#es) = (if matches-ex-entry P C pc e
    then Some e
    else match-ex-table-e P C pc es)

```

lemma *match-ex-table-e-sim*:

$(\text{match-ex-table-e } P \ C \ pc \ et = \text{Some } e) \implies (\text{match-ex-table } P \ C \ pc \ et = \text{Some } (\text{snd } (\text{snd } (\text{snd } e))))$

proof (*induct et*)

assume *A*: $(\text{match-ex-table-e } P \ C \ pc \ [] = \text{Some } e)$

from *A*

show $(\text{match-ex-table } P \ C \ pc \ [] = \text{Some } (\text{snd } (\text{snd } (\text{snd } e))))$

by *simp*

next

fix *a et*

assume *hyp*: $(\text{match-ex-table-e } P \ C \ pc \ et = \text{Some } e) \implies (\text{match-ex-table } P \ C \ pc \ et = \text{Some } (\text{snd } (\text{snd } (\text{snd } e))))$

assume *A*: $\text{match-ex-table-e } P \ C \ pc \ (a \# et) = \text{Some } e$

show $\text{match-ex-table } P \ C \ pc \ (a \# et) = \text{Some } (\text{snd } (\text{snd } (\text{snd } e)))$

proof (*cases matches-ex-entry P C pc a*)

case *True*

from *True A* **show** *?thesis*

by *simp*

next

case *False*

from *False A hyp* **show** *?thesis*

by *simp*

qed

qed

lemma *match-ex-table-e-sim2*:

$match\text{-}ex\text{-}table\ P\ C\ pc\ et = Some\ pc\text{-}h \implies (\exists\ e.(match\text{-}ex\text{-}table\text{-}e\ P\ C\ pc\ et = Some\ e) \wedge snd\ (snd\ (snd\ e)) = pc\text{-}h)$

proof (*induct et*)

assume *A*: ($match\text{-}ex\text{-}table\ P\ C\ pc\ [] = Some\ pc\text{-}h$)

from *A*

show $\exists\ e.\ match\text{-}ex\text{-}table\text{-}e\ P\ C\ pc\ [] = Some\ e \wedge snd\ (snd\ (snd\ e)) = pc\text{-}h$

by *simp*

next

fix *a et*

assume *hyp*: ($match\text{-}ex\text{-}table\ P\ C\ pc\ et = Some\ pc\text{-}h$) $\implies (\exists\ e.\ match\text{-}ex\text{-}table\text{-}e\ P\ C\ pc\ et = Some\ e \wedge snd\ (snd\ (snd\ e)) = pc\text{-}h)$

assume *A*: $match\text{-}ex\text{-}table\ P\ C\ pc\ (a\ \#\ et) = Some\ pc\text{-}h$

show $(\exists\ e.\ match\text{-}ex\text{-}table\text{-}e\ P\ C\ pc\ (a\ \#\ et) = Some\ e \wedge snd\ (snd\ (snd\ e)) = pc\text{-}h)$

proof (*cases matches-ex-entry P C pc a*)

case *True*

from *True A* **show** *?thesis*

by *simp*

next

case *False*

from *False A hyp* **show** *?thesis*

by *simp*

qed

qed

consts $match\text{-}ex\text{-}table::'m\ prog \Rightarrow cname \Rightarrow pc \Rightarrow ex\text{-}table \Rightarrow pc\ option$

defs $match\text{-}ex\text{-}table\text{-}def\ [simp]:$

$match\text{-}ex\text{-}table\ P\ C\ pc\ et == (case\ (JVMExceptions.match\text{-}ex\text{-}table\ P\ C\ pc\ et)\ of\ None \Rightarrow None\ |\ Some\ h \Rightarrow Some\ (fst\ h))$

consts

$succsXpt::(jbc\text{-}prog \times cname \times pos\ list) \Rightarrow (pos \times expr)\ list$

recdef $succsXpt\ measure\ (\lambda(\Pi,X,ps).\ length\ (domC\ \Pi) - (length\ ps))$

$succsXpt\ ((P,An),X,ps) = (case\ length\ (domC\ (P,An)) \leq length\ ps$

of *True* $\Rightarrow map\ (\lambda p.\ (p,TT))\ (domC\ (P,An))$

| *False* $\Rightarrow (case\ ps\ of\ [] \Rightarrow map\ (\lambda p.\ (p,TT))\ (domC\ (P,An))$

| $p\ \#\ pss \Rightarrow let\ (C,M,pc)=p;\ et = ex\text{-}table\text{-}of\ P\ C\ M;\ A=aF\ (P,An)\ p$

in ($case\ match\text{-}ex\text{-}table\text{-}e\ P\ X\ pc\ et$

of *None* $\Rightarrow concat\ (map\ (\lambda p'.\ succsXpt\ ((P,An),X,p'\ \#\ ps))\ (callers\ (P,An)\ p))$

| *Some e* $\Rightarrow (let\ (f,t,X',pc',d) = e$

in $[(C,M,pc'),And\ ((if\ pss=[]\ then\ []\ else\ [Catch\ X'\ A,Catch\ X\ (Pos$

$p))]$ @

$[xcpt\text{-}cond\ (P,An)\ X\ (last\ ps)]))])$)

constdefs

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succsExcept::jbc-prog ⇒ pos ⇒ (pos × expr) list
succsExcept Π p ≡ (case cmd Π p of None ⇒ []
| Some c ⇒ (case c
of Load n ⇒ []
| Store n ⇒ []
| Push v ⇒ []
| New C ⇒ succsXpt (Π,OutOfMemory,[p])
| Getfield F C ⇒ succsXpt (Π,NullPointer,[p])
| Putfield F C ⇒ succsXpt (Π,NullPointer,[p])
| Checkcast C ⇒ succsXpt (Π,ClassCast,[p])
| Invoke M n ⇒ succsXpt (Π,NullPointer,[p])
| Return ⇒ []
| Pop ⇒ []
| IBin no ⇒ []
| Goto t ⇒ []
| CmpEq ⇒ []
| IfIntCmp ro t ⇒ []
| IfFalse t ⇒ []
| Throw ⇒ succsXpt (Π,NullPointer,[p]) @ (case anF Π p of None ⇒ []
| Some a ⇒ concat (map (λ tp. (case tp of Void ⇒ [] | Boolean ⇒ [] | Integer ⇒ [] | NT ⇒
[]
| Class X ⇒ succsXpt (Π,X,[p]))) (extractTy (a,St 0))))
))

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constdefs addPos::pos ⇒ ((pos × expr) list) ⇒ (pos × expr) list
addPos p es ≡ (map (λ (p',B). (p',And [Pos p,B])) es)

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constdefs succsF::jbc-prog ⇒ pos ⇒ (pos × expr) list
succsF Π p ≡ addPos p (succsNormal Π p @ succsExcept Π p)

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1.2 Static Semantics

constdefs

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handlesEx'::jvm-prog ⇒ pos ⇒ cname list
handlesEx' P p ≡ remdups' (concat (map (λ(C,(S,Fs,Ms)).
      concat (map (λ(M,Ts,T,(mxs,mxl,is,et)).
        concat (map (λ(b,e,cn,h,d).
          if p=(C,M,h)
          then [cn] else [])
          et))
        Ms))
      P))

```

constdefs

handlesEx::jvm-prog \Rightarrow *pos* \Rightarrow *cname option*
handlesEx P p \equiv (case *handlesEx' P p* of [] \Rightarrow *None*
| *cn#cns* \Rightarrow *Some cn*)

constdefs

catchesEx::jvm-prog \Rightarrow *cname* \Rightarrow *pos* \Rightarrow *bool*
catchesEx P X p \equiv (let (*C,M,pc*) = *p*; *m* = *match-ex-table P X pc* (*ex-table-of P C M*)
in (case *m* of *None* \Rightarrow *False* | *Some pc'* \Rightarrow *True*))

constdefs *sys-xcpt-of* :: *instr* \Rightarrow *cname*

sys-xcpt-of i \equiv (case *i* of *New C* \Rightarrow *OutOfMemory*
| *Getfield F C* \Rightarrow *NullPointer*
| *Putfield F C* \Rightarrow *NullPointer*
| *Checkcast C* \Rightarrow *ClassCast*
| *Invoke M n* \Rightarrow *NullPointer*
| *Throw* \Rightarrow *NullPointer*
| - \Rightarrow *Exception*)

constdefs

$wpF :: jbc\text{-}prog \Rightarrow pos \Rightarrow pos \Rightarrow expr \Rightarrow expr$

$wpF \Pi p p' Q \equiv$

$(let\ pm = map\ (\lambda q. (Pos\ q, if\ q = p' \ then\ Pos\ p\ else\ FF))\ (getPosEx\ Q))\ in$

$(case\ cmd\ \Pi\ p\ of\ None \Rightarrow FF \ | \ Some\ ins \Rightarrow$

$(case\ handlesEx\ (fst\ \Pi)\ p'\ of\ None \Rightarrow (case\ ins$

$of\ Load\ n \Rightarrow substE\ (pm@(\map\ (\lambda k. (St\ k, if\ k = 0 \ then\ Rg\ n$
 $else\ St\ (k - 1)))\ (stkIds\ Q))))\ Q$

$| \ Store\ n \Rightarrow substE\ (pm@((Rg\ n, St\ 0) \#$
 $map\ (\lambda k. (St\ k, St\ (k+1)))\ (stkIds\ Q)))\ Q$

$| \ Push\ v \Rightarrow substE\ (pm@(\map\ (\lambda k. (St\ k, if\ k = 0 \ then\ Cn\ v$
 $else\ St\ (k - 1)))\ (stkIds\ Q)))\ Q$

$| \ New\ Cl \Rightarrow (let\ em = (pm@(\map\ (\lambda k. (St\ k, if\ k = 0 \ then\ NewA\ 0$
 $else\ St\ (k - 1)))\ (stkIds\ Q)))@$
 $(map\ (\lambda n. (NewA\ n, NewA\ (n+1)))\ (getNewEx\ Q)));$
 $gfe' = foldl\ (\lambda mp\ hex. (case\ hex$

$of\ GF\ F\ C\ ex \Rightarrow (let\ ex' = substE\ mp\ ex$
 $in\ (Gf\ F\ C\ ex, IF\ ex' \doteq\ NewA\ 0$
 $THEN\ Cn\ (the\ ((snd\ (blank\ (fst\ \Pi)\ Cl))(F, C)))$
 $ELSE\ Gf\ F\ C\ ex'))$

$| \ TY\ ex\ ty \Rightarrow (let\ ex' = substE\ mp\ ex$
 $in\ (Ty\ ex\ ty, IF\ ex' \doteq\ NewA\ 0$
 $THEN\ Cn\ (Bool\ ((Class\ Cl) = ty))$
 $ELSE\ Ty\ ex'\ ty)) \# mp)$
 $em\ (remdups'\ (getHeapEx\ Q))$
 $in\ substE\ gfe'\ Q)$

$| \ Getfield\ F\ C \Rightarrow substE\ (pm@[(St\ 0, Gf\ F\ C\ (St\ 0))]) \ Q$

$| \ Putfield\ F\ C \Rightarrow (let\ em = pm@(\map\ (\lambda k. (St\ k, St\ (k+2)))\ (stkIds\ Q));$
 $gfe' = foldl\ (\lambda mp\ ex. let\ ex' = substE\ mp\ ex$
 $in\ (Gf\ F\ C\ ex, IF\ (ex' \doteq\ St\ 1)$
 $THEN\ St\ 0\ ELSE\ Gf\ F\ C\ ex')) \# mp)$
 $em\ (remdups'\ (getGfEx\ F\ C\ Q))$
 $in\ substE\ gfe'\ Q)$

$| \ Checkcast\ C \Rightarrow substE\ pm\ Q$

$| \ Invoke\ M\ n \Rightarrow substE\ (pm@(FrNr, FrNr \oplus (Cn\ (Intg\ 1))) \#$
 $(map\ (\lambda k. (Rg\ k, if\ k \leq n \ then\ St\ (n-k)$
 $else\ (if\ k \leq n + fst\ (snd\ (snd\ (snd\ (snd\ (method\ (fst\ \Pi)\ (fst\ p')\ M))))$
 $then\ Cn\ arb$

$else\ none))\ (rgIds\ Q))\ @$
 $(map\ (\lambda k.\ (St\ k,\ none))\ (stkIds\ Q))\ @$
 $(map\ (\lambda ex.\ (Call\ ex,\ ex))\ (getCallEx\ Q))\ @$
 $(concat\ (map\ (\lambda(cn',\ ex').$
 $(if\ catchesEx\ (fst\ \Pi)\ cn'\ p$
 $then\ [(Catch\ cn'\ ex',\ ex')$
 $else\ [(Catch\ cn'\ ex',$
 $IF\ (FrNr\ \doteq\ Cn\ (Intg\ 1))\ THEN\ ex'$
 $ELSE\ Catch\ cn'\ ex'))\ (getCatchEx\ Q))\ Q$

| *Return* $\Rightarrow let\ (C,M,pc)=p;\ (P,An)=\Pi;\ n = length\ (fst\ (snd\ (method\ P\ C\ M)))$
 $in\ substE\ (pm@(FrNr,FrNr\ \ominus\ (Cn\ (Intg\ 1)))\ #$
 $(map\ (\lambda k.\ (St\ k,\ if\ 1 \leq k\ then\ Call\ (St\ (n+k))$
 $else\ St\ 0))\ (stkIds\ Q))\ @$
 $(map\ (\lambda k.\ (Rg\ k,\ Call\ (Rg\ k)))\ (rgIds\ Q))\ @$
 $(map\ (\lambda ex.\ (Call\ ex,\ Call\ (Call\ ex)))\ (getCallEx\ Q))\ @$
 $(map\ (\lambda(cn',\ ex').\ (Catch\ cn'\ ex',\ Call\ (Catch\ cn'\ ex'))$
 $(getCatchEx\ Q))\ Q$

| *Pop* $\Rightarrow substE\ (pm@(map\ (\lambda k.\ (St\ k,\ St\ (k+1)))\ (stkIds\ Q))\ Q$

| *IBin no* $\Rightarrow substE\ (pm@(map\ (\lambda k.\ (St\ k,\ if\ k=0\ then\ Num\ (St\ 1)\ no\ (St\ 0)\ else\ (St\ (k+1))))\ (stkIds\ Q))\ Q$

| *Goto t* $\Rightarrow substE\ pm\ Q$

| *CmpEq* $\Rightarrow substE\ (pm@(map\ (\lambda k.\ (St\ k,\ if\ k=0\ then\ (St\ 0)\ \doteq\ (St\ 1)$
 $else\ (St\ (k+1))))\ (stkIds\ Q))\ Q$

| *IfIntCmp ro t* $\Rightarrow substE\ (pm@(map\ (\lambda k.\ (St\ k,\ St\ (k+2)))\ (stkIds\ Q))\ Q$

| *IfFalse t* $\Rightarrow substE\ (pm@(map\ (\lambda k.\ (St\ k,\ St\ (k+1)))\ (stkIds\ Q))\ Q$

| *Throw* $\Rightarrow FF$)

| *Some cn* $\Rightarrow (let\ mp=pm@(map\ (\lambda k.\ (St\ k,\ if\ 1 \leq k\ then\ none$
 $else\ (if\ (ins = Throw)$
 $then\ (IF\ St\ 0 \doteq Cn\ (Null)$
 $THEN\ (Cn\ (Addr\ (addr-of-sys-xcpt\ NullPointer)))$
 $ELSE\ St\ 0)$
 $else\ Cn\ (Addr\ (addr-of-sys-xcpt\ (sys-xcpt-of\ ins))))\ (stkIds\ Q))\ @$
 $(let\ (C,M,pc)=p;\ (C',M',pc')=p';\ (P,An)=\Pi\ in$
 $(if\ match-ex-table\ P\ cn\ pc\ (ex-table-of\ P\ C\ M) = Some\ pc'\ then\ []$
 $else$
 $let\ rgm=map\ (\lambda k.\ (Rg\ k,\ Catch\ cn\ (Rg\ k)))\ (rgIds\ Q);$
 $om = map\ (\lambda ex.\ (Call\ ex,\ Catch\ cn\ (Call\ ex)))\ (getCallEx\ Q);$
 $cm = map\ (\lambda(cn',\ ex').\ (Catch\ cn'\ ex',\ Catch\ cn\ (Catch\ cn'\ ex'))$
 $(getCatchEx\ Q)$
 $in\ (FrNr,\ Catch\ cn\ FrNr)\ #rgm@om@cm))$

in substE mp Q))))

1.3 Wellformedness

constdefs $pTy::jvm\text{-}prog \Rightarrow ty_P$
 $pTy P \equiv (map\text{-}of2 (convert\text{-}pt (prog\text{-}kil P)))$

consts

$throwChk::jbc\text{-}prog \times instr\ option \times expr\ option \times pos \Rightarrow bool$

— Throw instructions require a type annotation (disjunction of Ty (St 0) (Class X)). The successor function uses these to find proper handlers. The initial instruction must not be Throw. In typesafe programs (no exception on top of initial stack) this cannot happen anyway.

defs $throwChk\text{-}def$:

$throwChk \equiv \lambda (\Pi, ins, an, p). (case\ ins\ of\ None \Rightarrow True$
 | $Some\ c \Rightarrow$
 ($case\ c\ of\ Throw \Rightarrow (case\ an\ of\ None \Rightarrow False$
 | $Some\ A \Rightarrow (if\ p=ipc\ \Pi\ then\ False\ else$
 ($case\ extractTy\ (A, St\ 0)\ of\ [] \Rightarrow False$
 | $ty\#tys \Rightarrow (list\text{-}all\ (\lambda\ tp.$
 ($case\ tp\ of\ Void \Rightarrow False\ | Boolean \Rightarrow False$
 | $Integer \Rightarrow False\ | NT \Rightarrow False$
 | $Class\ X \Rightarrow True))\ (ty\#tys))))))$
 | $- \Rightarrow True))$

lemma $throwChk\text{-}Throw\text{-}A$ [simp]:

$throwChk (\Pi, Some\ Throw, Some\ A, p) =$
 ($if\ p=ipc\ \Pi\ then\ False\ else$
 ($case\ extractTy\ (A, St\ 0)\ of\ [] \Rightarrow False$
 | $ty\#tys \Rightarrow (list\text{-}all\ (\lambda\ tp. (case\ tp\ of\ Void \Rightarrow False\ | Boolean \Rightarrow False$
 | $Integer \Rightarrow False\ | NT \Rightarrow False$
 | $Class\ X \Rightarrow True))\ (ty\#tys))))$)

lemma $throwChk\text{-}Throw\text{-}None$ [simp]:

$throwChk (\Pi, Some\ Throw, None, p) = False$

lemma $throwChk\text{-}oth\text{-}None$ [simp]:

$ins \neq Some\ Throw \Longrightarrow throwChk (\Pi, ins, None, p) = True$

lemma $throwChk\text{-}oth\text{-}Some$ [simp]:

$ins \neq Some\ Throw \Longrightarrow throwChk (\Pi, ins, Some\ A, p) = True$

consts

$invokeChk::jbc\text{-}prog \times (instr\ option) \times (expr\ option) \times pos \Rightarrow bool$

— Invoke instructions require a type annotation (disjunction of Ty (St n) (Class X)). The successor function uses these to determine the potential method entry points. In addition the first instruction must not be Invoke. In typesafe programs this cannot happen anyway (no object reference on top of initial stack). We also forbid recursive calls of the main method ($M = \text{fst} (\text{snd} (\text{ipc } \Pi))$). This ensures that the frame stack never becomes empty (A Return in method main stops execution).

defs *invokeChk-def*:

$$\begin{aligned} \text{invokeChk} \equiv & \lambda (\Pi, \text{ins}, \text{an}, p). (\text{case } \text{ins} \text{ of } \text{None} \Rightarrow \text{True} \mid \text{Some } c \Rightarrow \text{case } c \\ & \text{of } \text{Invoke } M \ n \Rightarrow (\text{case } \text{an} \text{ of } \text{None} \Rightarrow \text{False} \\ & \quad \mid \text{Some } A \Rightarrow (\text{if } p = \text{ipc } \Pi \text{ then } \text{False} \text{ else} \\ & \quad \quad (\text{case } \text{extractTy } (A, \text{St } n) \text{ of } [] \Rightarrow \text{False} \\ & \quad \quad \quad \mid \text{ty}\#\text{tys} \Rightarrow (\text{list-all } (\lambda \text{tp}. (\text{case } \text{tp} \text{ of } \text{Void} \Rightarrow \text{False} \mid \text{Boolean} \Rightarrow \text{False} \\ & \quad \quad \quad \quad \mid \text{Integer} \Rightarrow \text{False} \mid \text{NT} \Rightarrow \text{True} \\ & \quad \quad \quad \quad \mid \text{Class } X \Rightarrow \text{has-method } (\text{fst } \Pi) \ X \ M)) (\text{ty}\#\text{tys})) \wedge M \\ & \neq \text{fst} (\text{snd} (\text{ipc } \Pi)))))) \\ & \mid - \Rightarrow \text{True}) \end{aligned}$$

lemma *invokeChk-Invoke-A [simp]*:

$$\begin{aligned} \text{invokeChk } (\Pi, \text{Some } (\text{Invoke } M \ n), \text{Some } A, p) = & (\text{if } p = \text{ipc } \Pi \text{ then } \text{False} \text{ else} \\ & (\text{case } \text{extractTy } (A, \text{St } n) \text{ of } [] \Rightarrow \text{False} \mid \text{ty}\#\text{tys} \Rightarrow (\text{list-all } (\lambda \text{tp}. (\text{case } \text{tp} \text{ of } \text{Void} \Rightarrow \text{False} \mid \text{Boolean} \\ & \Rightarrow \text{False} \\ & \quad \mid \text{Integer} \Rightarrow \text{False} \mid \text{NT} \Rightarrow \text{True} \\ & \quad \mid \text{Class } X \Rightarrow \text{has-method } (\text{fst } \Pi) \ X \ M)) (\text{ty}\#\text{tys})) \wedge M \neq \text{fst} (\text{snd} (\text{ipc} \\ & \Pi)))) \end{aligned}$$

lemma *invokeChk-Invoke-None [simp]*:

$$\text{invokeChk } (\Pi, \text{Some } (\text{Invoke } M \ n), \text{None}, p) = \text{False}$$

lemma *invokeChk-oth-None [simp]*:

$$(\forall M \ n. \text{ins} \neq \text{Some } (\text{Invoke } M \ n)) \Longrightarrow \text{invokeChk } (\Pi, \text{ins}, \text{None}, p) = \text{True}$$

lemma *invokeChk-oth-Some [simp]*:

$$(\forall M \ n. \text{ins} \neq \text{Some } (\text{Invoke } M \ n)) \Longrightarrow \text{invokeChk } (\Pi, \text{ins}, \text{Some } A, p) = \text{True}$$

consts

$$\text{checkPos} :: \text{jbc-prog} \Rightarrow (\text{pos list}) \Rightarrow \text{bool}$$

— checkPos ensures that targets of backward jumps ($\text{idx} (\text{domC } \Pi) \ p' \neq \text{idx} (\text{domC } \Pi) \ p$) are annotated. This ensures termination of the generic VCG. In addition all successors must be in the code range ($p' \text{ mem } (\text{domC } \Pi)$) and successors from the normal successor function must not be entry points of handlers. Throw and Invoke instructions are checked for the extra requirements described above.

primrec

$$\begin{aligned} \text{checkPos } \Pi \ [] &= \text{True} \\ \text{checkPos } \Pi \ (p\#\text{ps}) &= (\text{if } (\text{let } \text{scsn} = \text{map } \text{fst} (\text{succsNormal } \Pi \ p); \\ & \quad \text{scse} = \text{map } \text{fst} (\text{succsExcept } \Pi \ p) \\ & \text{in } \text{list-all } (\lambda p'. ((\text{idx} (\text{domC } \Pi) \ p' \leq \text{idx} (\text{domC } \Pi) \ p) \end{aligned}$$

$$\begin{aligned}
& \longrightarrow \text{anF } \Pi \text{ } p' \neq \text{None}) \wedge p' \text{ mem } (\text{domC } \Pi) \wedge \\
& \quad (p' \text{ mem } \text{scsn} \longrightarrow \text{handlesEx } (\text{fst } \Pi) \text{ } p' = \text{None}) \wedge p' \neq \text{ipc } \Pi) \\
& (\text{scsn } @ \text{scse}) \wedge (\text{set } \text{scse} \subset \text{set } (\text{domC } \Pi)) \wedge \text{throwChk } (\Pi, \text{cmd } \Pi \text{ } p, \text{anF } \Pi \\
p, p) \wedge \\
& \quad \text{invokeChk } (\Pi, \text{cmd } \Pi \text{ } p, \text{anF } \Pi \text{ } p, p) \\
& \text{then } (\text{checkPos } \Pi \text{ } ps) \text{ else False}
\end{aligned}$$

lemma *checkPos-split*:

$$\text{checkPos } \Pi \text{ } (l1 @ l2) = ((\text{checkPos } \Pi \text{ } l1) \wedge (\text{checkPos } \Pi \text{ } l2))$$

constdefs *checkExTables* :: *jdbc-prog* \Rightarrow *bool*

$$\begin{aligned}
\text{checkExTables } \Pi \equiv & \text{list-all } (\lambda x. x) (\text{concat } (\text{map } (\lambda(C, (S, Fs, Ms)). \\
& \quad \text{concat } (\text{map } (\lambda(M, Ts, T, (mxs, mxl, is, et)). \\
& \quad \quad (\text{map } (\lambda(b, e, cn, h, d). d = 0 \wedge \\
& \quad \quad \quad (C, M, h) \in \text{set } (\text{domC } \Pi) \wedge \\
\text{remdups}' (\text{concat } (\text{map } (\lambda(b, e, cn, h', d). \text{if } h' = h \text{ then } [cn] \text{ else } []) \text{ et})) = [cn] \text{ et})) \\
& \quad Ms)) \\
& (\text{fst } \Pi)))
\end{aligned}$$

— Programs are wellformed iff (1) all positions are wellformed (*checkPos*). That is - targets of backward jumps are annotated (enforces VCG termination). - Throw and Invoke instructions have type annotations (possibly inserted by the bytecode verifier). - the main method is not invoked (only automatically at start up). (2) all exception tables are wellformed. That is - remaining stack height *d* is 0 (no catch inside expressions). - the catching class is not *Object*. - handler entry points are within the code range. - handler entry points are distinct for each handler. (3) all classes and methods have distinct names. (4) the programs contains all system classes (*object*, *exceptions*). (5) the initial position is in the code range. (6) the main method has no arguments (type safety theorems from BV require this).

constdefs

wf :: *jdbc-prog* \Rightarrow *bool*

$$\begin{aligned}
\text{wf } \Pi \equiv & \text{checkPos } \Pi \text{ } (\text{domC } \Pi) \wedge \\
& \text{checkExTables } \Pi \wedge \\
& \text{distinct } (\text{classnames } (\text{fst } \Pi)) \wedge \\
& \text{distinct } (\text{methodnames } (\text{fst } \Pi)) \wedge \\
& (\exists \text{cdl. } \text{fst } \Pi = (\text{SystemClasses } @ \text{cdl})) \wedge (\text{ipc } \Pi \in \text{set } (\text{domC } \Pi)) \\
& \wedge \text{wf-jvm-prog-phi } (pTy (\text{fst } \Pi)) (\text{fst } \Pi) \\
& \wedge \text{fst } (\text{snd } (\text{method } (\text{fst } \Pi) (\text{fst } (\text{ipc } \Pi)) (\text{fst } (\text{snd } (\text{ipc } \Pi)))))) = []
\end{aligned}$$

1.4 System Invariants

The following functions yield formulas that hold for all states reachable in wellformed programs

1.5 Position information

constdefs *inv-Pos::jbc-prog* \Rightarrow *pos* \Rightarrow *expr*
inv-Pos Π *p* \equiv *Pos p*

1.6 Frame Stack Size

constdefs *inv-FrNr::jbc-prog* \Rightarrow *pos* \Rightarrow *expr*
inv-FrNr Π \equiv ($\lambda(C,M,pc). \text{if } (\text{let } (C0,M0,pc0) = \text{ipc } \Pi \text{ in } C=C0 \wedge M=M0) \text{ then Eq FrNr } (Cn \text{ (Intg 1)}) \text{ else } (\text{Rel } (Cn \text{ (Intg 1)}) \text{ Less FrNr}))$)

1.7 System Exception Types

constdefs *inv-ExTys::jbc-prog* \Rightarrow *pos* \Rightarrow *expr*
inv-ExTys Π *p* \equiv *And* [*Ty* (*Cn* (*Addr* (*addr-of-sys-xcpt* *NullPointer*))) (*Class* *NullPointer*),
Ty (*Cn* (*Addr* (*addr-of-sys-xcpt* *ClassCast*))) (*Class* *ClassCast*),
Ty (*Cn* (*Addr* (*addr-of-sys-xcpt* *OutOfMemory*))) (*Class* *OutOfMemory*)]

1.8 Bytecode Verifier Types

constdefs *conv-st::jvm-prog* \Rightarrow *ty_i* \Rightarrow *expr*
conv-st *P* \equiv ($\lambda(st,rt). (\text{let } ex\text{-}st = \text{map } (\lambda n. STy P (St n) (st!n)) (\text{upt } 0 \text{ (length } st));$
 $ex\text{-}rt = \text{map } (\lambda n. (\text{case } rt!n \text{ of } Err \Rightarrow \text{not-none } (Rg n)$
 $\quad \quad \quad | OK \text{ } tp \Rightarrow STy P (Rg n) tp)) (\text{upt } 0 \text{ (length } rt))$
 $\text{in } (And (ex\text{-}st @ ex\text{-}rt))))$)

constdefs *annotate-types::jvm-prog* \Rightarrow (*(cname* \times *mname)* \times (*ty_i' list*)) *list* \Rightarrow (*pos* \Rightarrow *expr*)
annotate-types *P* *pt* \equiv ($\lambda(C,M,pc).$
 $(\text{if } (C,M) \text{ mem } (\text{methodnames } P) \text{ then } (\text{case } pt ? (C,M) \text{ of } None \Rightarrow FF$
 $\quad | \text{Some } mt \Rightarrow (\text{if } pc < \text{length } mt \text{ then } (\text{case } mt ! pc \text{ of } None \Rightarrow FF | \text{Some } ty_i \Rightarrow \text{conv-st } P \text{ } ty_i)$
 $\quad \quad \quad \text{else } TT))$
 $\text{else } TT))$)

constdefs *inv-Ty::jbc-prog* \Rightarrow *pos* \Rightarrow *expr*
inv-Ty Π *p* \equiv *annotate-types* (*fst* Π) (*convert-pt* (*prog-kil* (*fst* Π))) *p*

1.9 Instantiating the VCG

constdefs
vcg-jbc $::$ *jbc-prog* \Rightarrow *expr*
vcg-jbc *prg* \equiv *vcG* *And* *Imp* *FF* *ipc* *initF* *safeF* *succsF* *wpF* *domC* *domA* *anF* *prg*

1.10 Upgrade the VCG

Here we upgrade the VCG by instantiating it with successor functions that add invariants to

branch conditions.**constdefs** $upg::(jbc\text{-}prog \Rightarrow pos \Rightarrow expr) \Rightarrow (jbc\text{-}prog \Rightarrow pos \Rightarrow (pos \times expr) list)$
 $\Rightarrow (jbc\text{-}prog \Rightarrow pos \Rightarrow (pos \times expr) list)$
 $upg\ iF\ sucF \equiv (\lambda\ \Pi\ p.\ map\ (\lambda\ (p',B).\ (p',\ And\ [B,\ iF\ \Pi\ p]))\ (sucF\ \Pi\ p))$

1.11 Upgrade Frame Stack Size

constdefs $succsFrNrF:: jbc\text{-}prog \Rightarrow pos \Rightarrow (pos \times expr) list$
 $succsFrNrF \equiv upg\ inv\text{-}FrNr\ succsF$

constdefs

$vcgFrNr :: jbc\text{-}prog \Rightarrow expr$
 $vcgFrNr\ prg \equiv vcG\ And\ Imp\ FF\ ipc\ initF\ safeF\ succsFrNrF\ wpF\ domC\ domA\ anF\ prg$

1.12 Upgrade System Exception Types.

constdefs $succsExTysF:: jbc\text{-}prog \Rightarrow pos \Rightarrow (pos \times expr) list$
 $succsExTysF \equiv upg\ inv\text{-}ExTys\ succsFrNrF$

constdefs

$vcgExTys :: jbc\text{-}prog \Rightarrow expr$
 $vcgExTys\ prg \equiv vcG\ And\ Imp\ FF\ ipc\ initF\ safeF\ succsExTysF\ wpF\ domC\ domA\ anF\ prg$

1.13 Upgrade Types

constdefs $succsTyF:: jbc\text{-}prog \Rightarrow pos \Rightarrow (pos \times expr) list$
 $succsTyF \equiv upg\ inv\text{-}Ty\ succsExTysF$

constdefs

$vcgTy :: jbc\text{-}prog \Rightarrow expr$
 $vcgTy\ \Pi \equiv (vcG\ And\ Imp\ FF\ ipc\ initF\ safeF\ succsTyF\ wpF\ domC\ domA\ anF\ \Pi)$

end